Direct numerical simulation of high Reynolds number turbulence by the K computer

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Joint work with
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JAPAN-RUSSIA WORKSHOP ON
SUPERCOMPUTER MODELING, INSTABILITY AND TURBULENCE
IN FLUID DYNAMICS (JR SMIT2015) March 6, 2015

High Reynolds number turbulence

- High Re turbulence（⟺ Low Re turbulence）

High Re

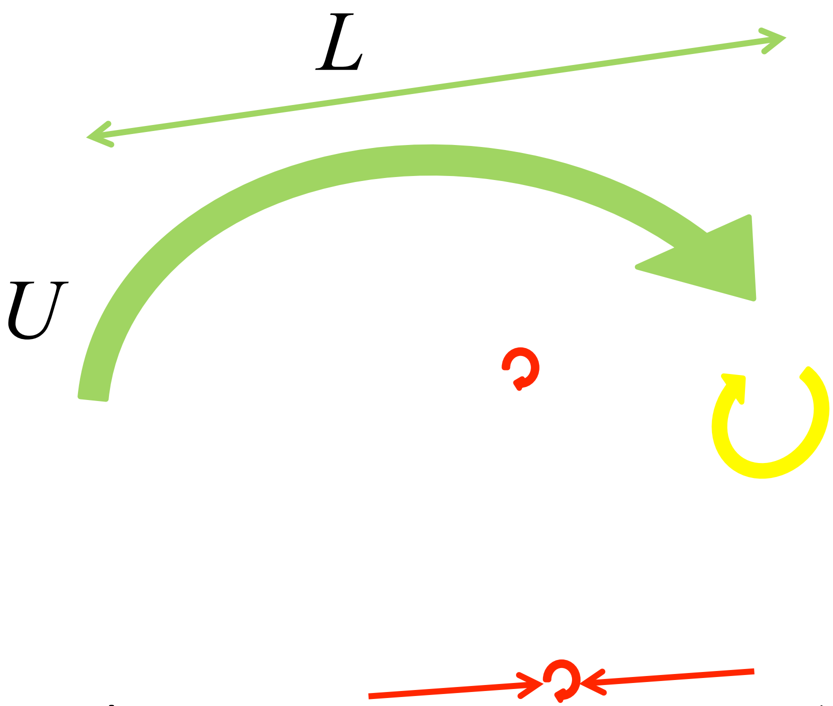
Low Re

The higher the Reynolds numbers,
the larger the scale ratio.

Direct Numerical Simulation(DNS) of turbulence

- To understand the nature of high Re turbulence

DNS of the Navier-Stokes equations is very useful.



Scale ratio

$$\frac{L}{\eta} \sim \mathrm{Re}^{3/4}$$

Cf. Kolmogorov's theory

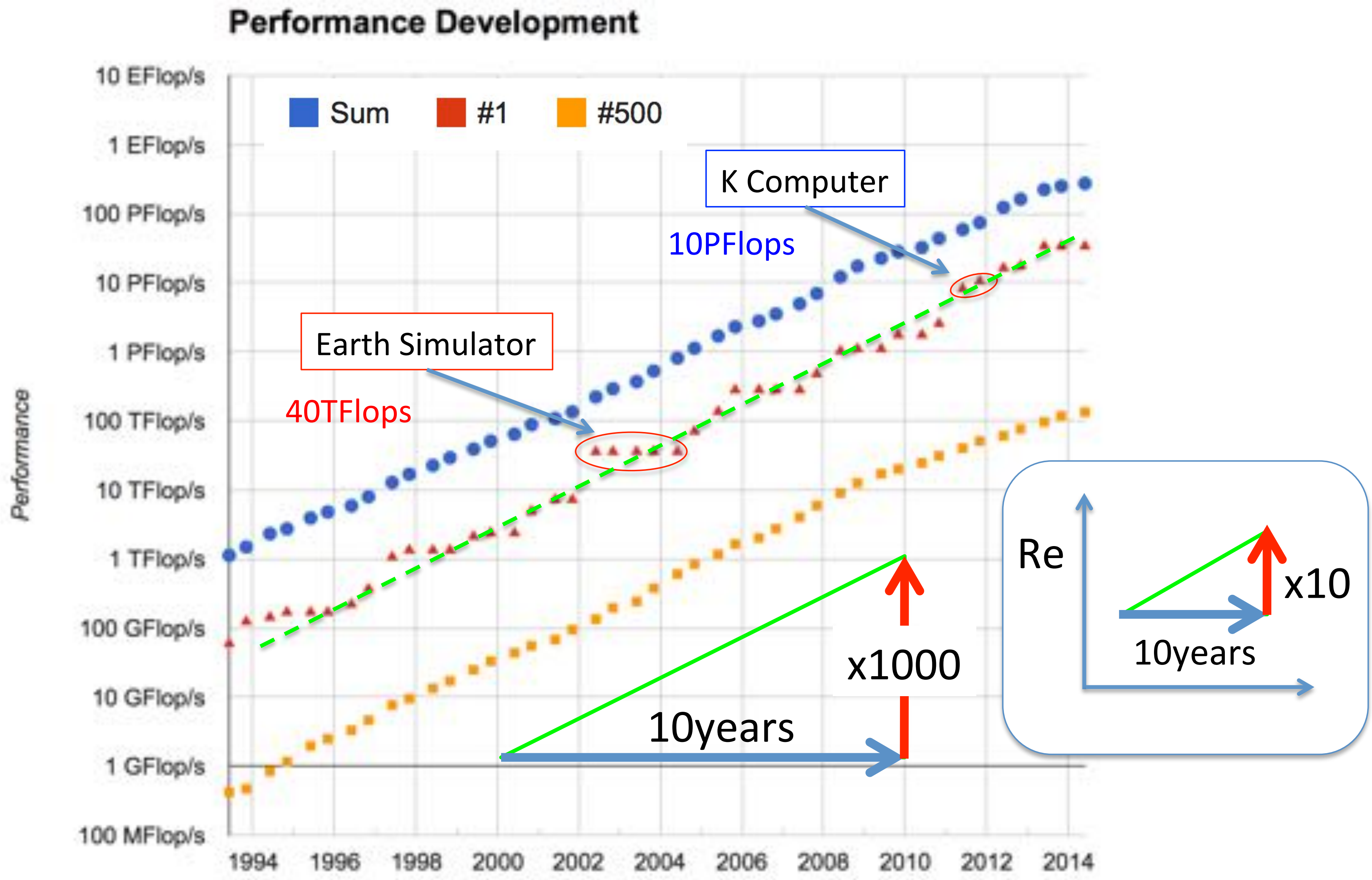
Degree of Freedom $\sim \left(\frac{L}{\eta}\right)^3 \sim \mathrm{Re}^{9/4}$

Re=UL/ν

$$\eta = (\nu^3 / \bar{\varepsilon})^{1/4}$$

Computational Cost $\sim \mathrm{Re}^3$

Recent development of supercomputers

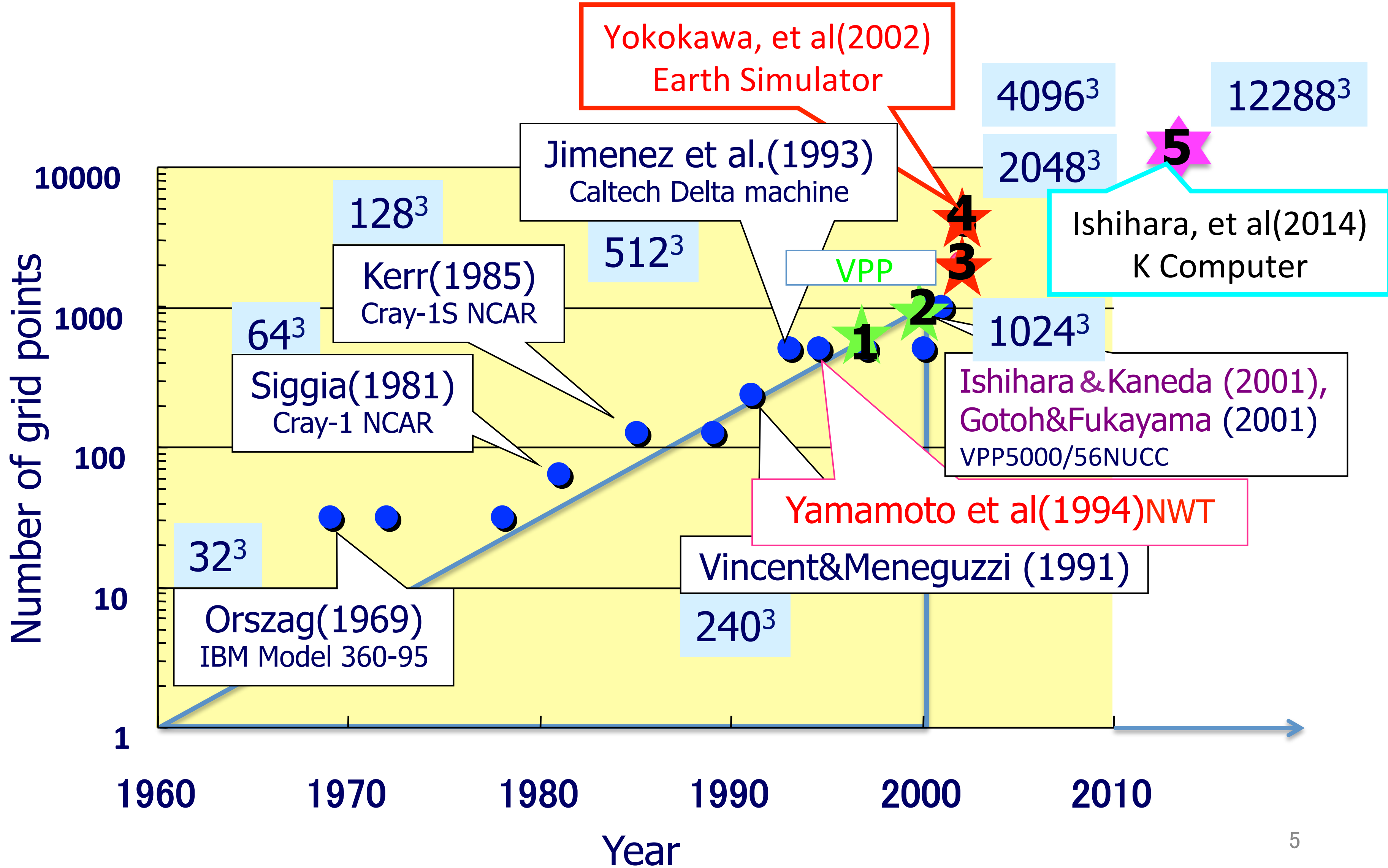


http://www.top500.org/statistics/perfdevel/

History of representative DNS

Homogeneous Isotropic Turbulence under periodic BC

Challenge for higher Re



In this talk

- Direct Numerical Simulation of high Re turbulence
  - Review of the DNS on Earth Simulator (up to 4096^3)
  - New results by the DNS on K computer (up to 12288^3)

DNS of box turbulence using Earth Simulator

40TFlops @ 640 node

- Fourier Spectral method

(Yokokawa et al 2002, Kaneda et al 2003)

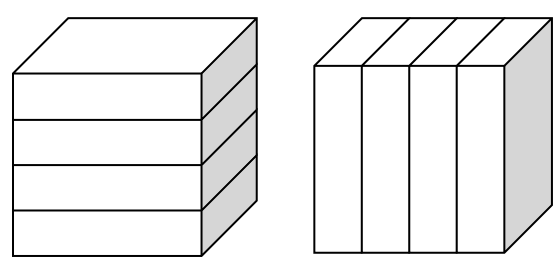
$$\left(\frac{\partial}{\partial t} + \nu k^2 - c(k)\right)\hat{\mathbf{u}}(\mathbf{k}) = \mathbf{P}(\mathbf{k}) \cdot \widehat{(\mathbf{u} \times \mathbf{w})}(\mathbf{k})$$

$$\mathbf{k} \cdot \hat{\mathbf{u}}(\mathbf{k}) = 0 \qquad \mathbf{w} = \nabla \times \mathbf{u}$$

Forcing: negative viscosity (to keep the total energy constant)

$$c(k) = \begin{cases} c & (k < 2.5) \\ 0 & \text{otherwise} \end{cases}$$

- 1D decomposition



16.4TFlops @512 node

Efficiency is 50%!!

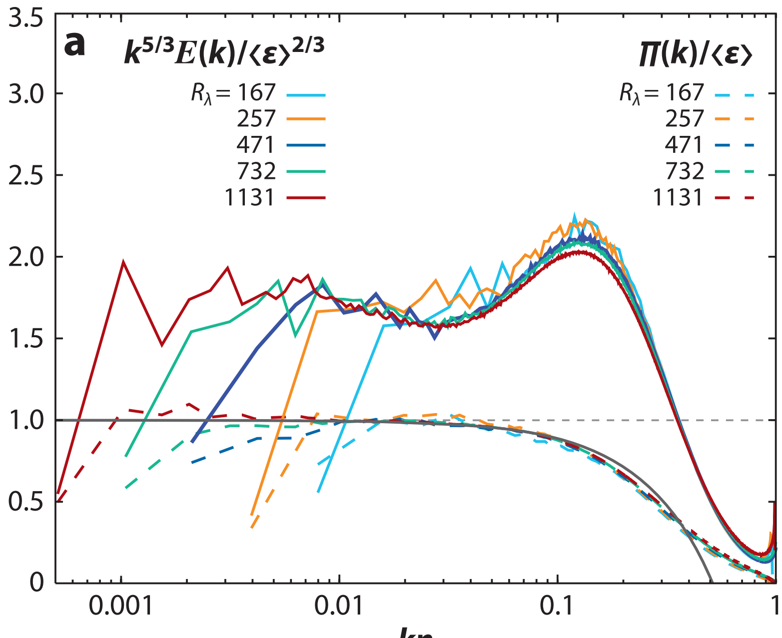
Two series of DNS using ES

- Series 1($k_{max}\eta$ =1)
- Series 2($k_{max}\eta$ =2)

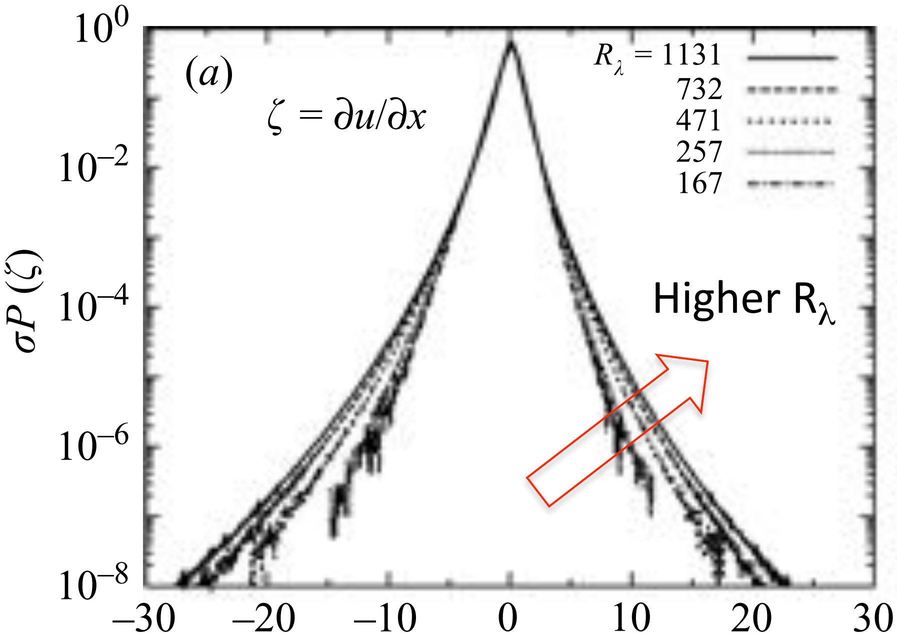
N	256^3	512^3	1024^3	2048^3	4096^3
R_λ (Series 1)	167	257	471	732	1131
R_λ (Series 2)	94	173	268	429	675

The higher the Reynolds numbers, the wider the inertial range

The higher the Reynolds numbers, the stronger the intermittency



I, Gotoh, Kaneda ARF2009



I, Kaneda, et al. JFM2007

Energy Spectrum in the Inertial Range

- K41

$$E(k) = C_K \langle \varepsilon \rangle^{2/3} k^{-5/3}, \quad \Pi(k) \equiv \int_k^\infty T(k')\,dk' = \langle \varepsilon \rangle$$

- DNS of Turbulence in a Periodic Box with 4096^3 Grid Points

$$E(k) \sim \langle \varepsilon \rangle^{2/3} k^{-5/3-\mu}, \quad \mu \sim 0.10$$

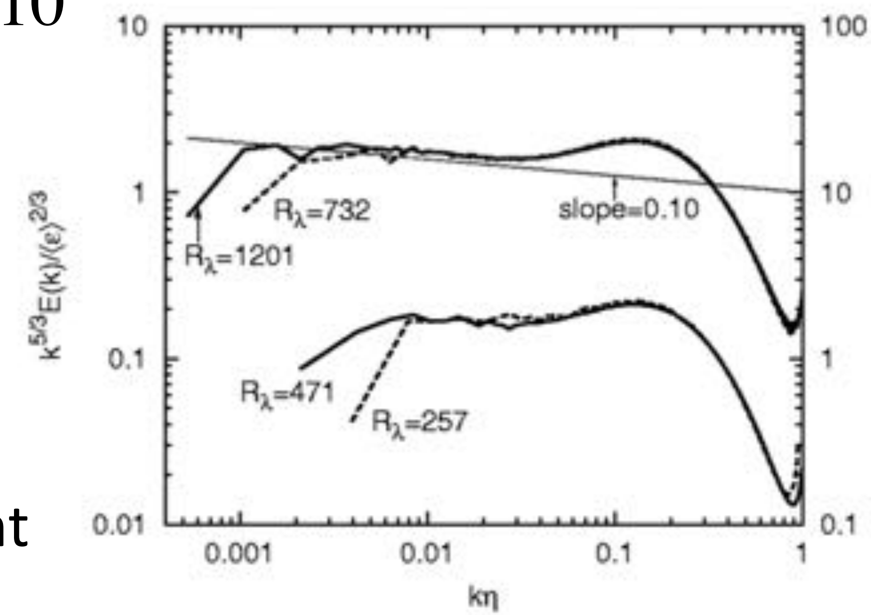
$$\Pi(k) \sim \langle \varepsilon \rangle$$

- Similar slope
  - Hyper Viscosity Simulation Haugen & Brandenburg (2004)
  - DNS with 4096^3 grid points Donzis & Sreenivasan JFM (2010)

Interpretation of the exponent

… not yet

Kaneda et al. PF (2003)

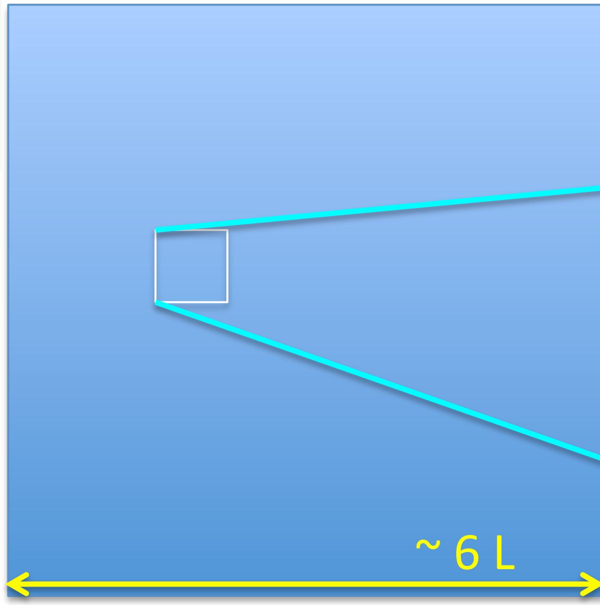


Significant, high vorticity, intermittent structure

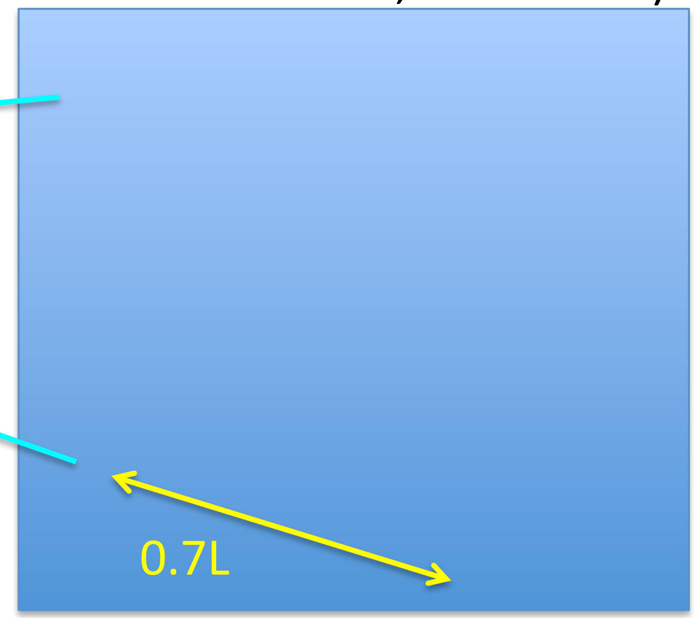
Re=3.6x10^4
R_λ =*O*(10^3)

λ/η= 66
L/λ=32

ARF2009



Complex thin-shear layers
(Ishihara Kaneda Hunt, FTAC 2013)

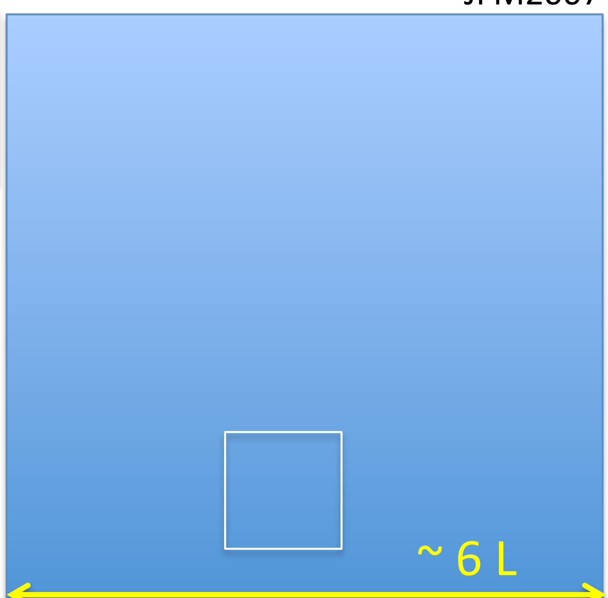


Width ~ O(L), Thickness ~ O(λ) >> η

Re=3.2x10^2
R_λ =*O*(10^2)

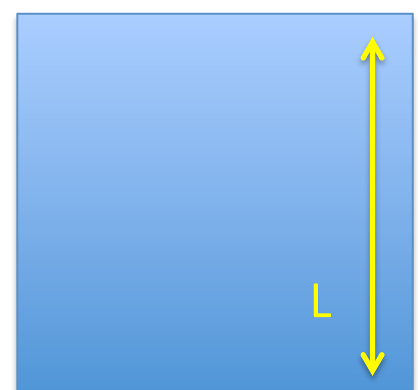
λ/η= 20
L/λ=3

JFM2007



Isolated vortices

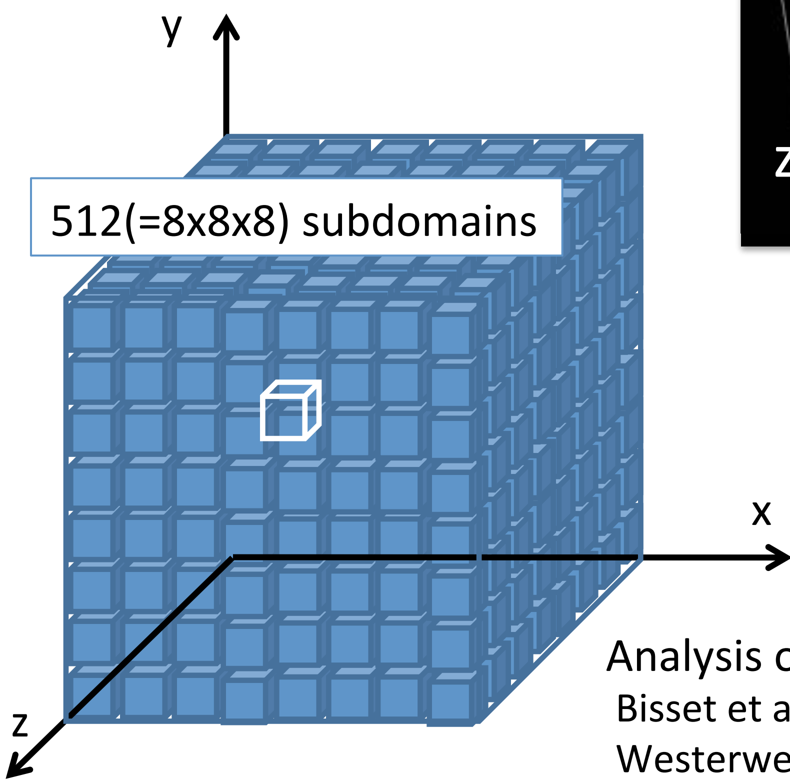
Length ~ O(L)
Thickness ~ O(10η)
~ O(λ)



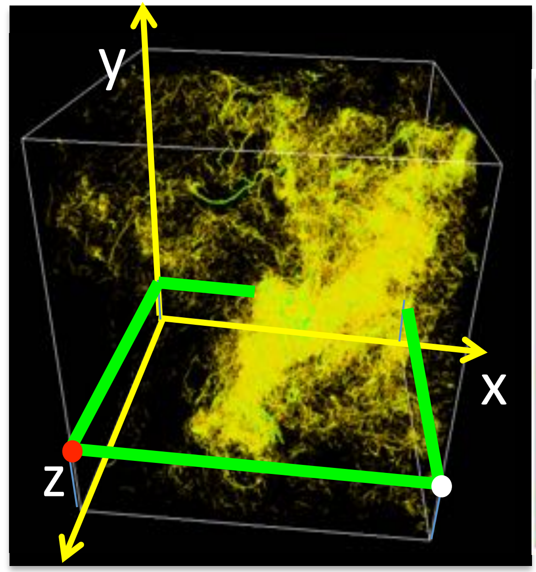
Analysis in IKH2013

Based on one snapshot data of DNS of forced NS eq. with 4096^3 grid points (Kaneda et al, 2003)

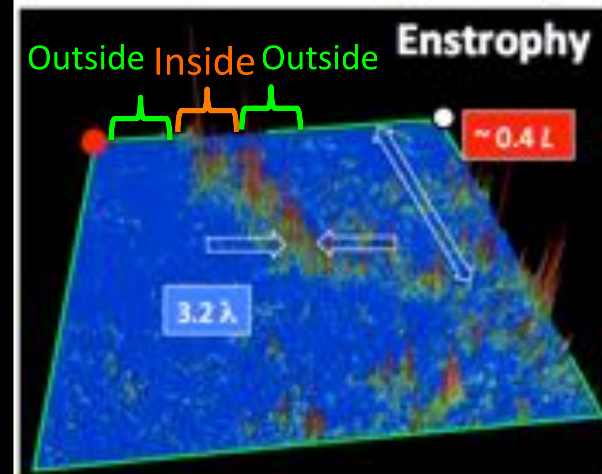
$k_{max}\eta=1$, $R_\lambda=1131$

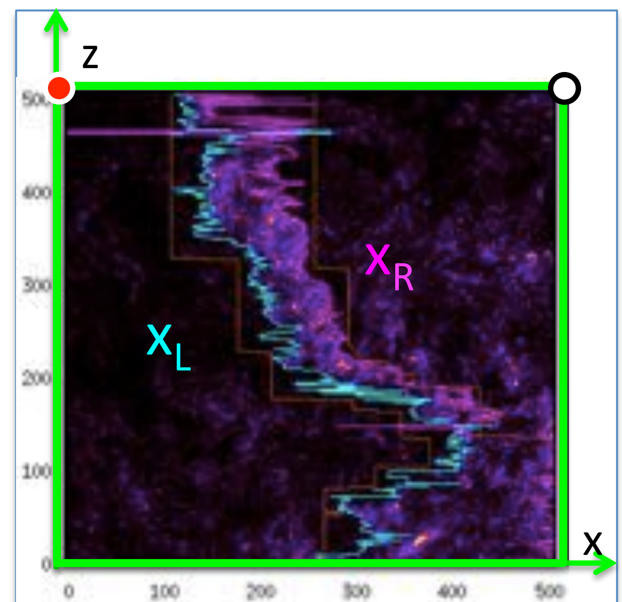


One of active (high enstrophy) sub-domains



Enstrophy distribution on a cross section





Analysis of Conditional statistics

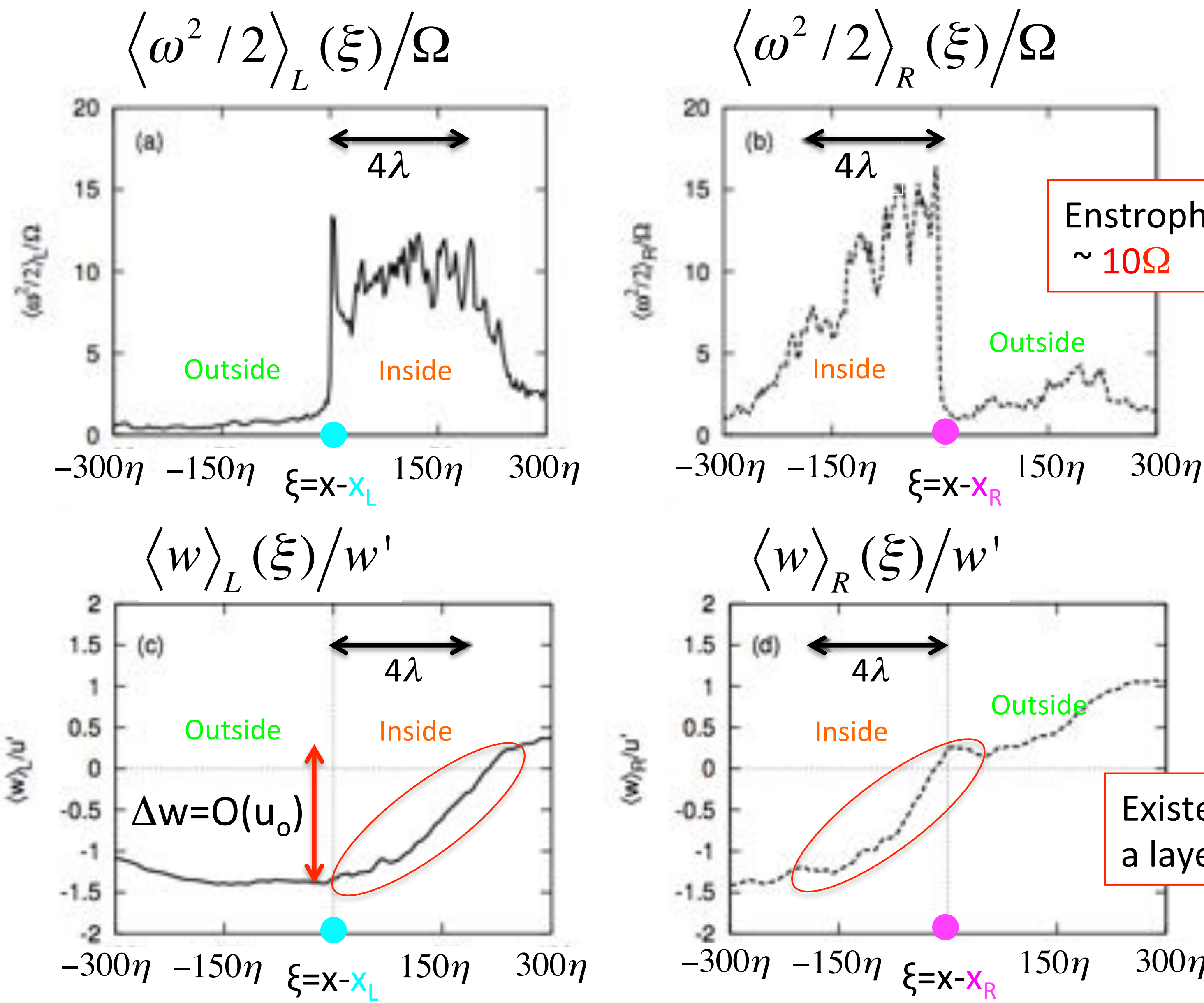
Bisset et al 2002

Westerweel et al 2009

da Silva & dos Reis 2011

(Ishihara Kaneda Hunt, FTAC 2013)

Conditional averages

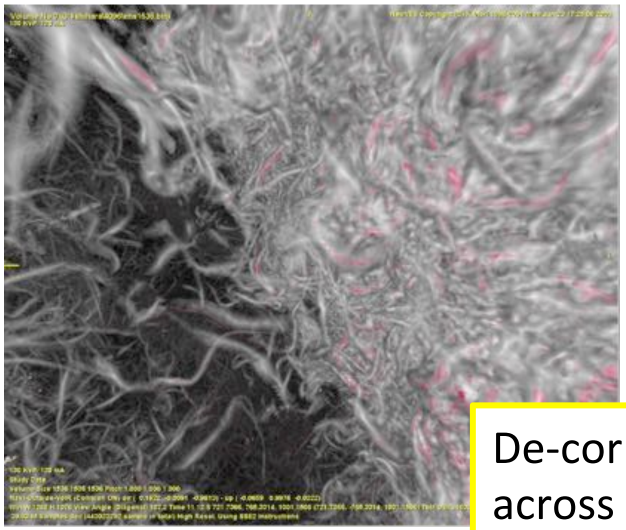


Enstrophy in the layer
~ 10Ω

Existence of
a layer-scale shear

Sharp interface

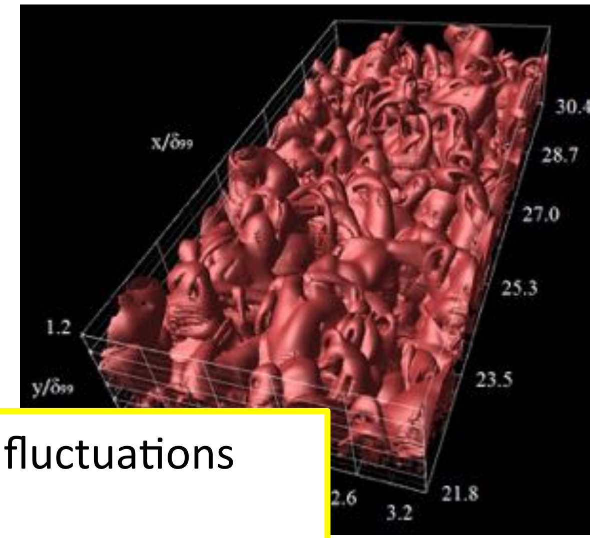
Sharp interface of internal thin shear layer (IKH2013)



$$\omega = 2\left\langle \omega^2 \right\rangle^{1/2}$$

$$\omega = 6\left\langle \omega^2 \right\rangle^{1/2}$$

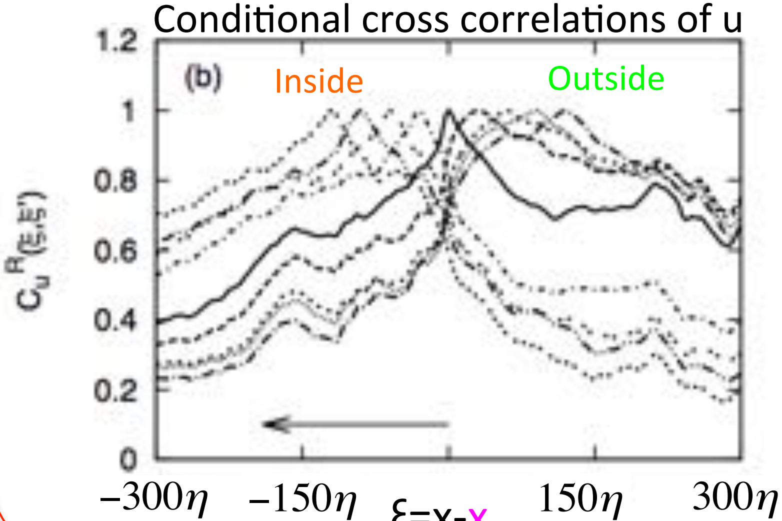
T/NT interface of external TBL (I,Ogasawara,Hunt2014)



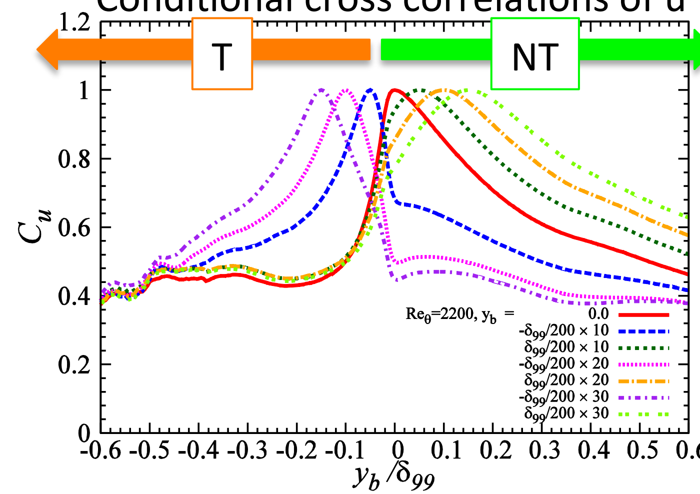
De-correlation of the velocity fluctuations across the sharp interface

- Blocking mechanism (Hunt & Durbin 1999)

Conditional cross correlations of u

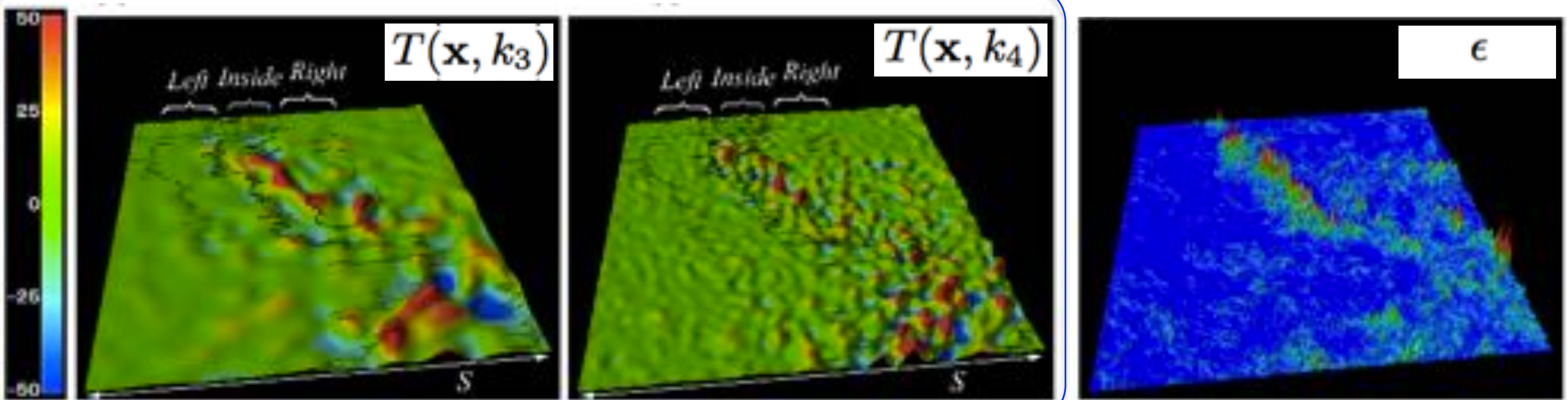


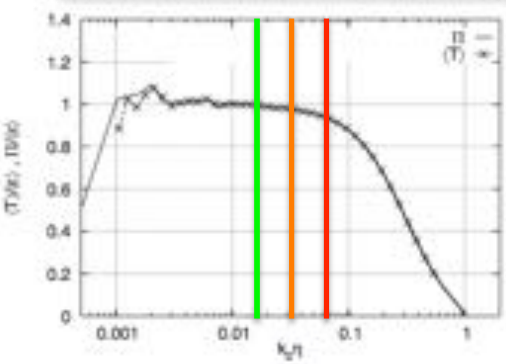
Conditional cross correlations of u



(Ishihara Kaneda Hunt, FTAC 2013)

Energy transfer $T(\mathbf{x},k)$ and energy dissipation ε near the layer





Large amplitude positive/negative (i.e. downscale/upscale) fluctuations of T near the thin layer

$\langle T(x,k)\rangle_{Inside} = \langle\varepsilon\rangle_{Inside} \sim 10\,\langle\varepsilon\rangle$ for $k > \pi/l$

l :thickness of the layer

$10 \sim L/l \sim R_\lambda/100$

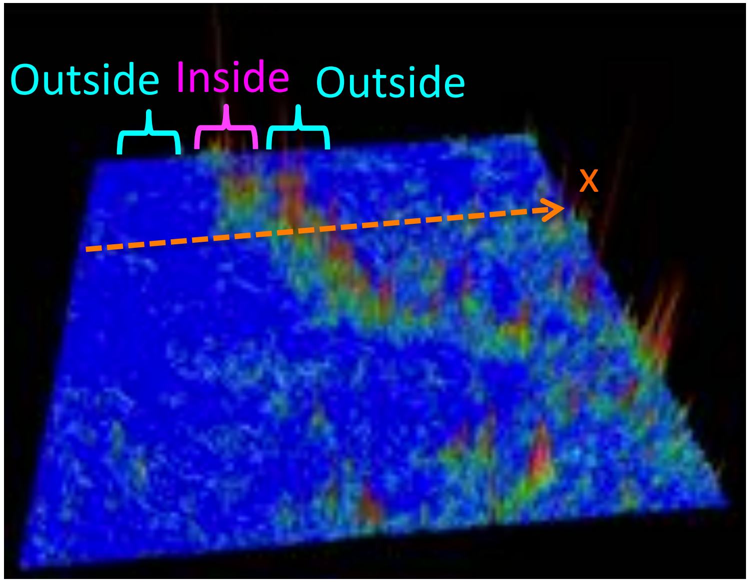
A net energy flux from the larger scale motions from outside

A		$\langle A\rangle/\langle\epsilon\rangle$	$\langle A\rangle_{Left}/\langle\epsilon\rangle$	$\langle A\rangle_{Inside}/\langle\epsilon\rangle$	$\langle A\rangle_{Right}/\langle\epsilon\rangle$
$T(\mathbf{x},k_2)$	$\pi/k_2 \approx 2.9\lambda$	0.99(3.86)	3.76(5.99)	3.9(12.4)	1.2(19.7)
$T(\mathbf{x},k_3)$	$\pi/k_3 \approx 1.4\lambda$	0.98(4.24)	0.36(2.17)	10.7(22.5)	5.7(18.5)
$T(\mathbf{x},k_4)$	$\pi/k_4 \approx 0.7\lambda$	0.94(4.93)	1.03(3.55)	10.2(24.6)	4.0(13.6)
ϵ	—	1	0.88(1.40)	10.2(11.9)	2.44(3.49)

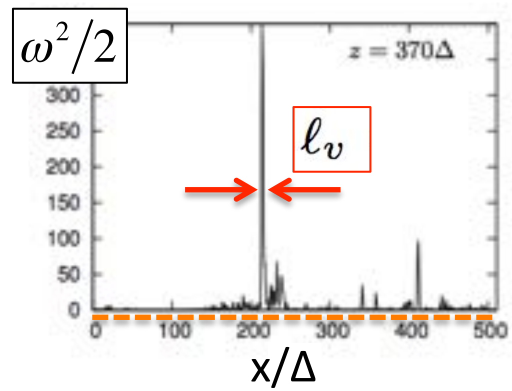
(Ishihara Kaneda Hunt, FTAC 2013)

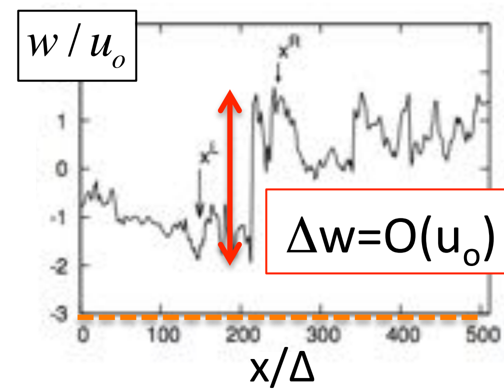
Inside structure of the shear layers

Distribution of the strong vortices inside the layer



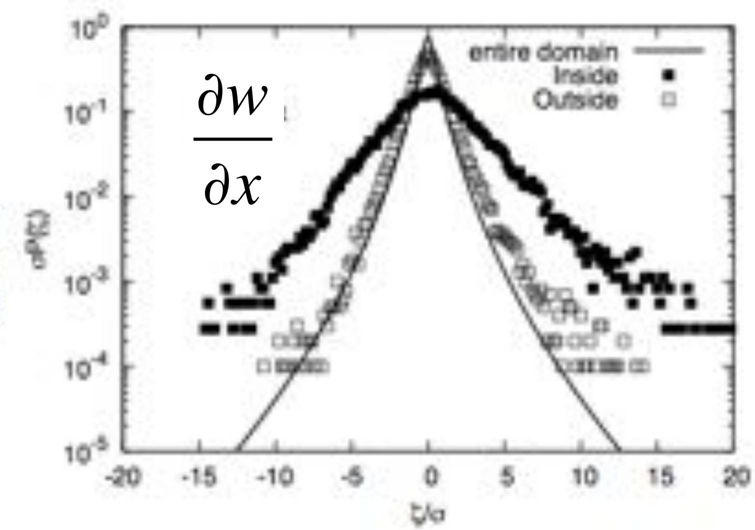
($\Delta=2\pi/4096\sim3\eta$: grid spacing)





Thickness of the micro-scale vortices: $\ell_v \sim 10\eta$

(insensitive to their strength)



Very strong vorticity of $O(u_o/10\eta)$

$\gg u_{Kol}/\eta = 1/\tau_{Kol}$ (K41)

Velocity jump of $O(u_o)$ over distances of $O(10\eta)$

$\gg u_{Kol} \sim u_o Re^{-1/4}$ (K41)

The layers may dominate the extreme point values of the statistical distributions of dissipation, velocity and vorticity fluctuations

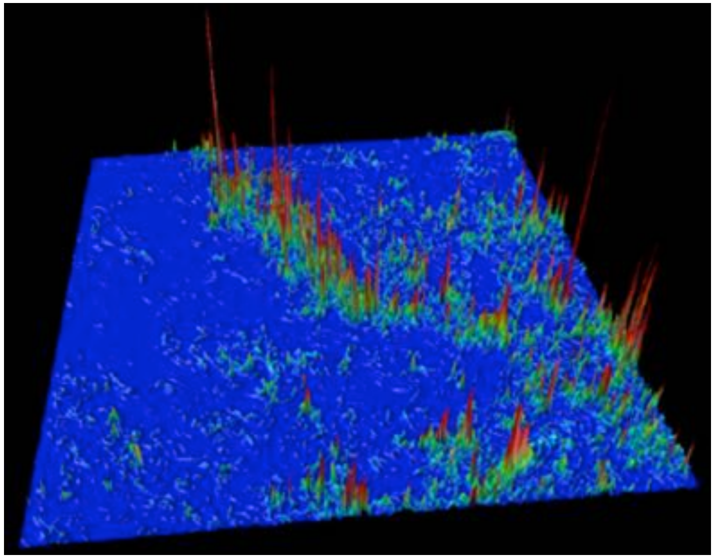
High enstrophy regions
in coarse-grained data

Large clusters of the connected high enstrophy regions

Large clusters of the connected high enstrophy regions

(Ishihara Kaneda Hunt, FTAC 2013)

Characteristics of the thin shear layers



- Strong vortices are close packed and dense in the layers
- Thickness=O($\lambda \propto Re^{-1/2}L$), Size =O(L), Distance=O(L)
- Strong shear across the layers (velocity jump of O(u_o))
- Extreme events(high velocity jump, high vorticity) in the layers
- Act as a barrier by blocking and filtering the velocity fluctuations
- High energy dissipation and high energy transfer within the layers
- Large fluctuation of the energy transfer (+ and -)

DNS of turbulence in a periodic box on K computer

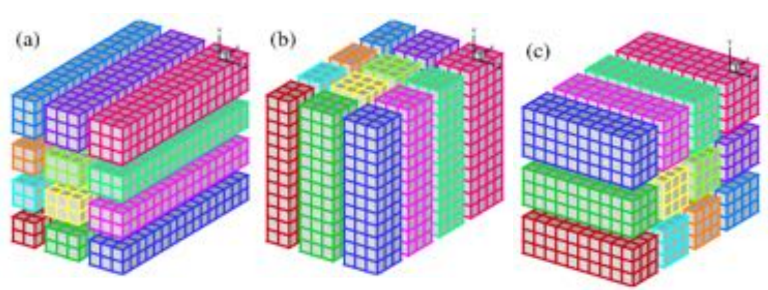
11.28PFlops @ 88128 node

- The same spectral method as used for Earth Simulator

Forcing: negative viscosity (to keep the total energy constant)

$$c(k) = \begin{cases} c & (k < 2.5) \\ 0 & \text{otherwise} \end{cases}$$

- A 2D decomposition code for the K computer



N	# of nodes	TFlops	efficiency
6144	96x64	30.20	3.84%
8192	128x64	32.93	3.14%
12288	192x128	70.46	2.24%
12288	384x128	128.3	2.04%

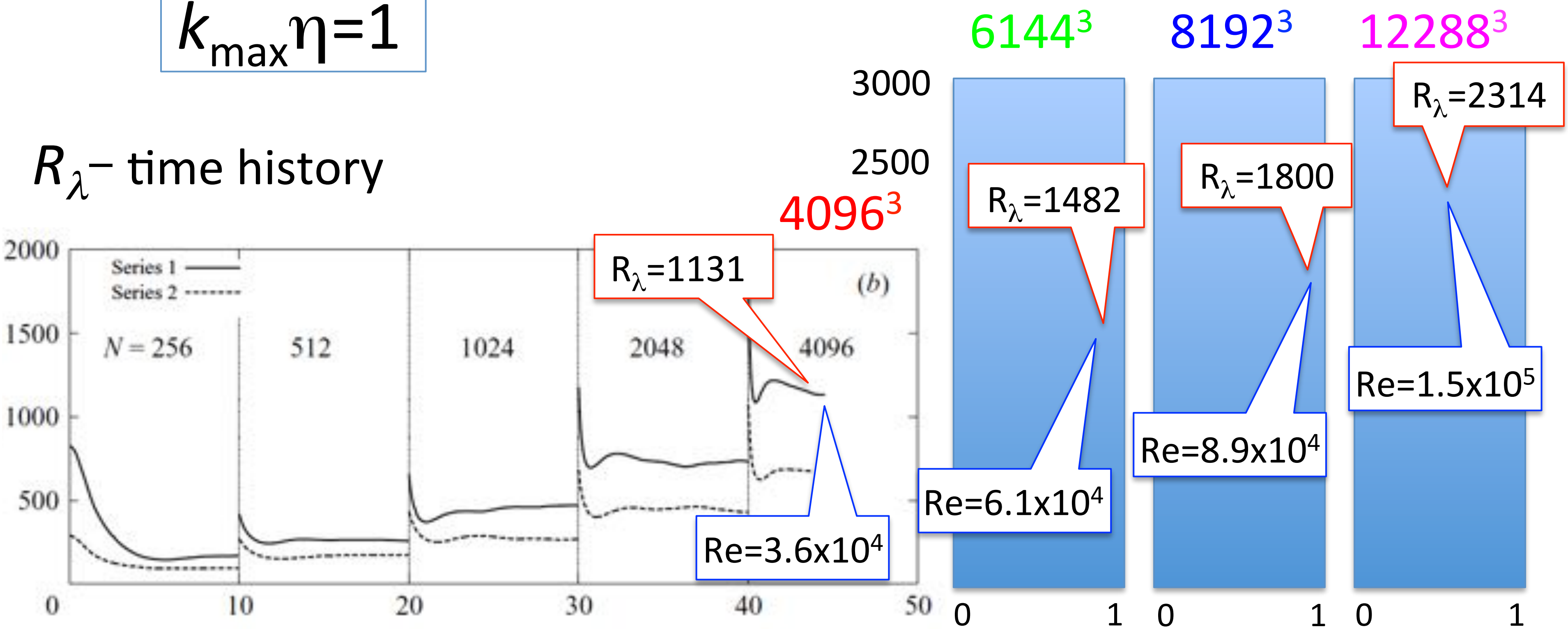
(double precision)

Efficiency of 50% was obtained on ES using 512 nodes.

DNS with $6144^3/8192^3/12288^3$ grid points

$k_{max}\eta=1$

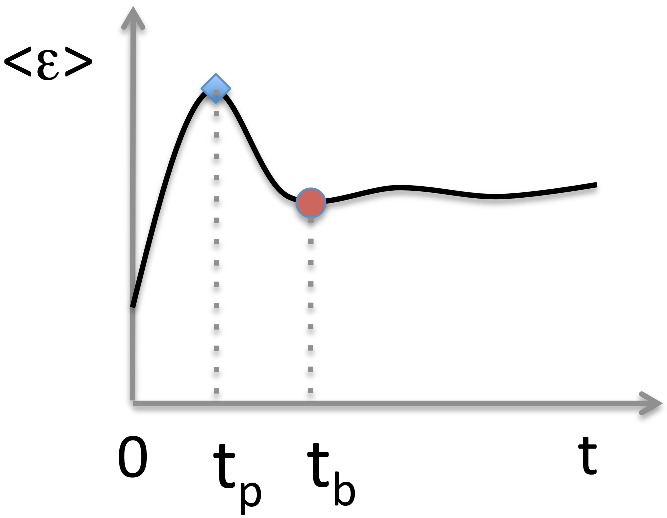
R_λ – time history



(Ishihara et al JFM 2007)

$T=L/u_o\sim2$

Time dependence of <ε>



$$R_\lambda = \frac{u'\lambda}{\nu} = \frac{\sqrt{15}u'^2}{\left(\nu\langle\varepsilon\rangle\right)^{1/2}}$$

In our case
Initial condition: a developed state
Forcing: negative viscosity

$t_p \sim 25 t_\eta \sim 6.45(\lambda/u') \propto T/R_\lambda$

$t_b \sim 80 t_\eta \sim 20.7(\lambda/u') \propto T/R_\lambda$

Time development of energy spectrum and energy flux (from the 8192^3 DNS)

$k^{5/3}E(k)L^{2/3}/u'^2$

$\Pi(k)L/u'^3$

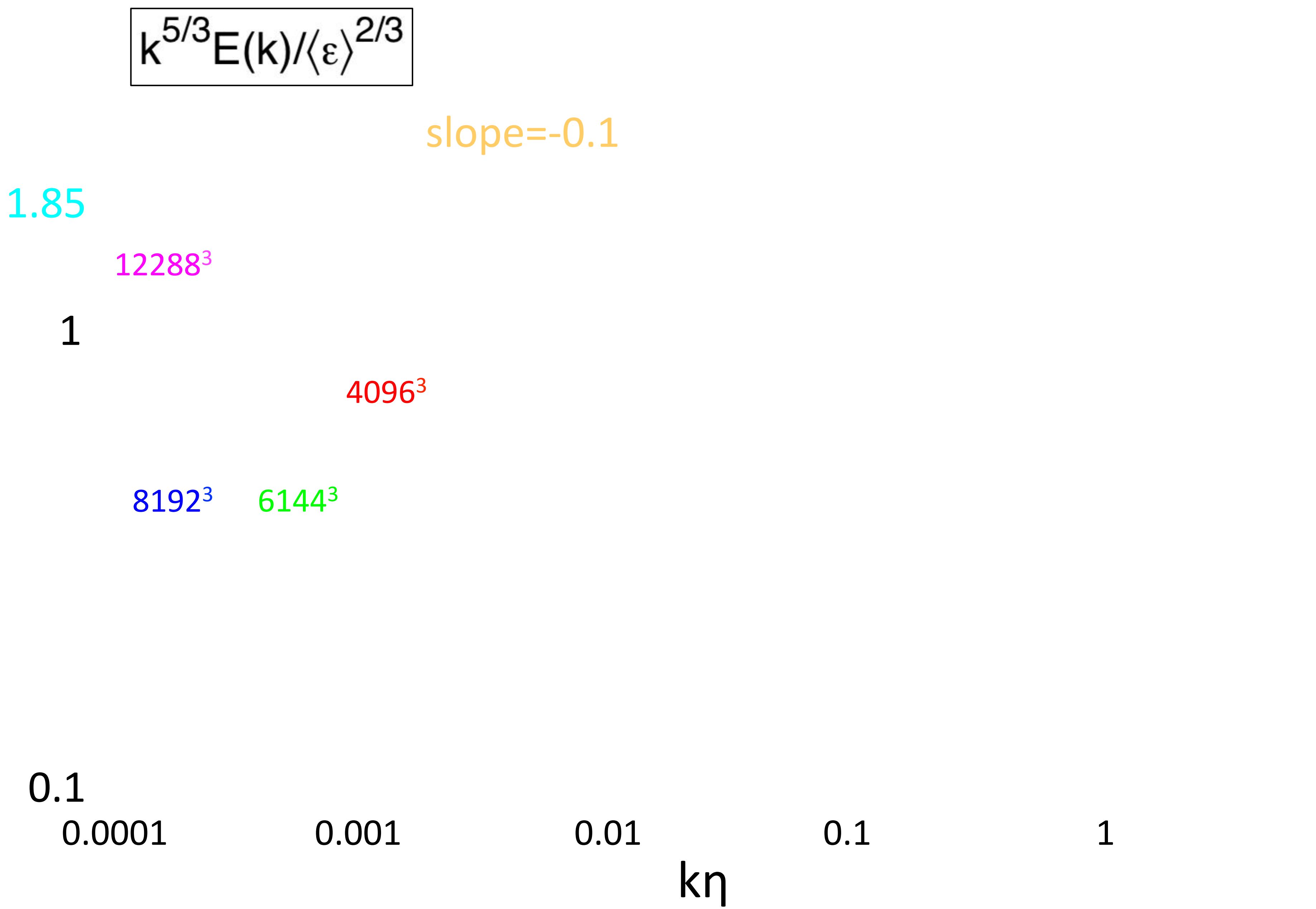
kL

$k^{5/3}E(k)/\langle\varepsilon\rangle^{2/3}$

$\Pi(k)/\langle\varepsilon\rangle$

$k\eta$

Average vs. snapshots (from 4096^3 DNS)


$k^{5/3}E(k)/\langle\varepsilon\rangle^{2/3}$
slope=-0.1
1.85
12288^3
1
4096^3
8192^3
6144^3
0.1
0.0001
0.001
0.01
0.1
1
kη

$k^{5/3}E(k)/\langle \varepsilon \rangle^{2/3}$

1

12288^3

6144^3

8192^3

4096^3

0.1

1 10 100 1000 10000

kL

Energy dissipation

$k^{5/3}E(k)/\langle\varepsilon\rangle^{2/3}$

$$D(k) = 2\nu\int_k^\infty k^2 E(k)dk$$

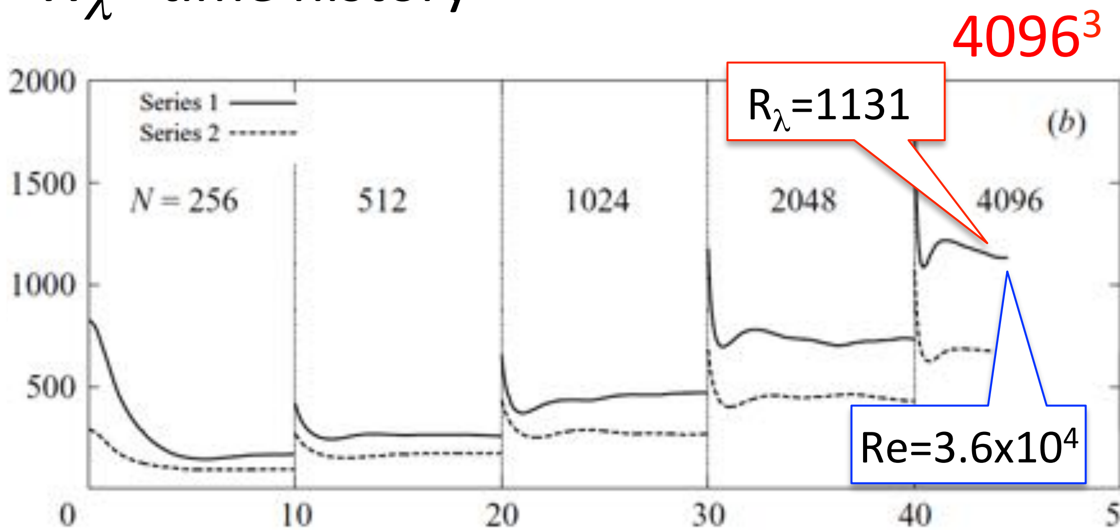
$$\frac{D(k)}{D(0)}$$

kη

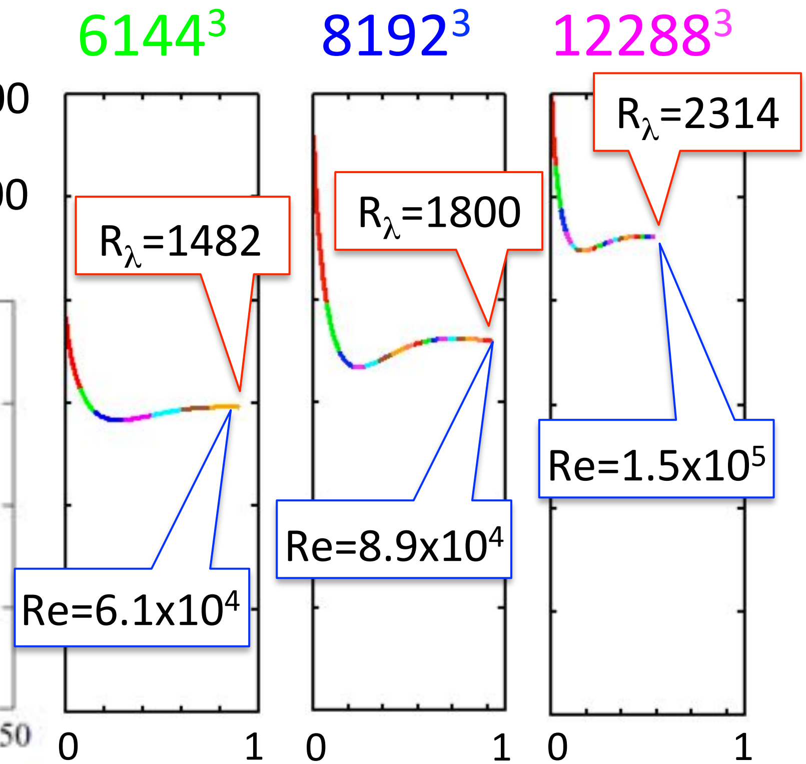
DNS with $6144^3/8192^3/12288^3$ grid points

$k_{max}\eta=1$

R_λ– time history

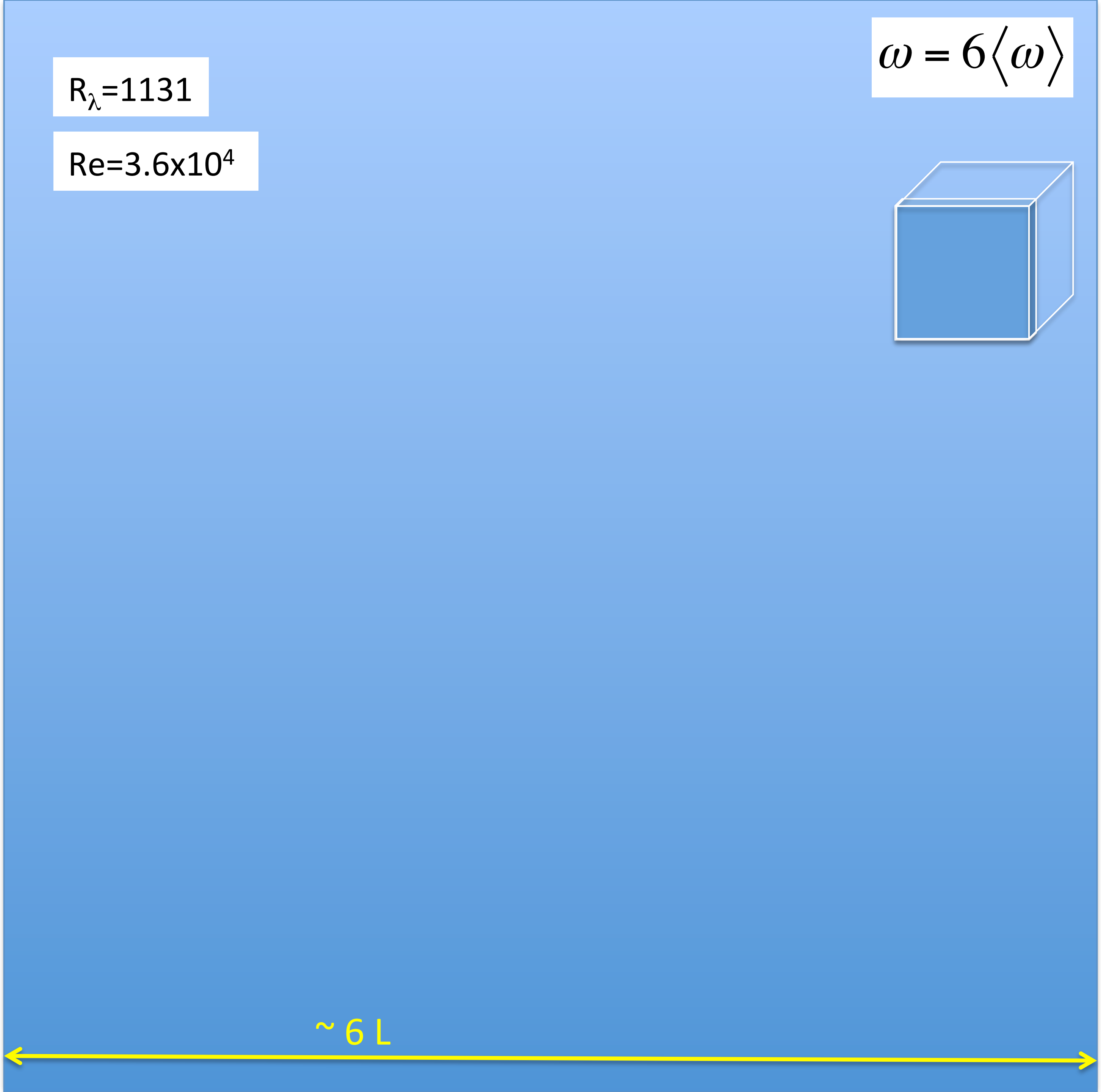


(Ishihara et al JFM 2007)

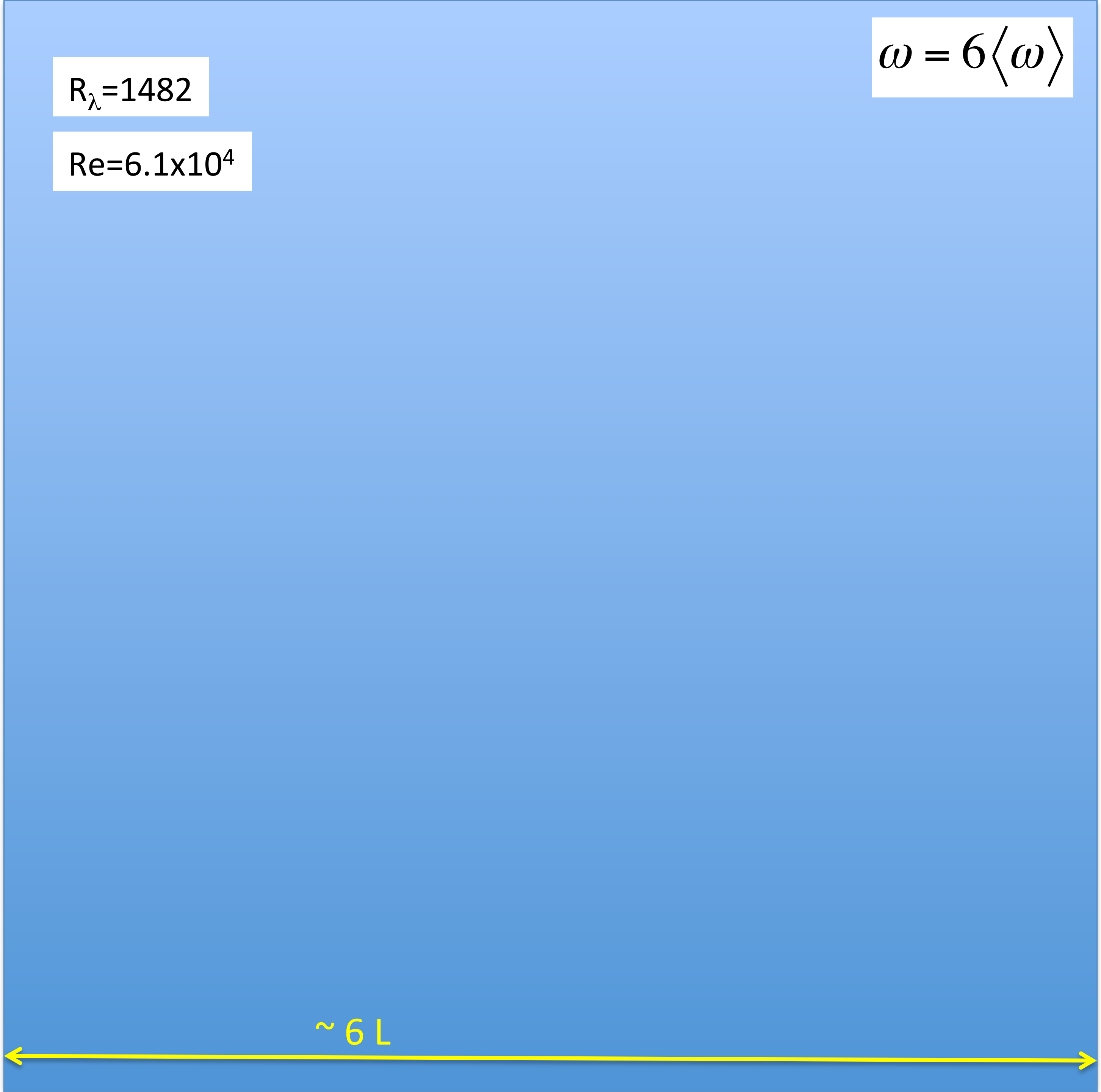


$T=L/u_o$~2

4096^3



6144^3



8192^3

R_λ=1800

Re=8.9×10^4

$$\omega = 6\langle\omega\rangle$$

~ 6 L

6144^3 $R_\lambda=1482$

$Re=6.1\times10^4$

8192^3 $R_\lambda=1800$

$Re=8.9\times10^4$

$$\omega = 6\langle\omega\rangle$$

Summary

- We have performed large-scale DNS of forced incompressible turbulence with up to 12288^3 grid points using K computer. Data-analysis and visualization show the following.
  - The maximum values of Re are $R_\lambda=2314$, $Re=1.5\times10^5$
  - All the energy spectra with different Reynolds numbers ($R_\lambda>1000$) have the wavenumber range of a steeper slope;

$$E(k) \sim k^{-5/3-\mu}, \ (\mu \sim 0.1)$$

  - The spectra are well normalized not by L but by η, at the wavenumber range of the steeper slope, where a few percent of energy is dissipated.

(The steeper slope is not inherent character in the inertial range, and is affected by viscosity.)

  - Thin shear layers (the layers in which strong vortices are close-packed and dense) become more common and important structures in higher Reynolds number turbulence.