Extraction of Hierarchical Vortices from Turbulent Field using Component-Dependent Filter

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Outline

1. **Motivation & Objective**

2. **Problem Setting & Previous Studies**
   + Target flow (Homogeneous isotropic turbulence)
   + Vortex reconstruction / filtering

3. **Turbulent Vortex Motion**
   + Vortex deformation (Stretching)
   + Geometric relation between multi-scale vortices

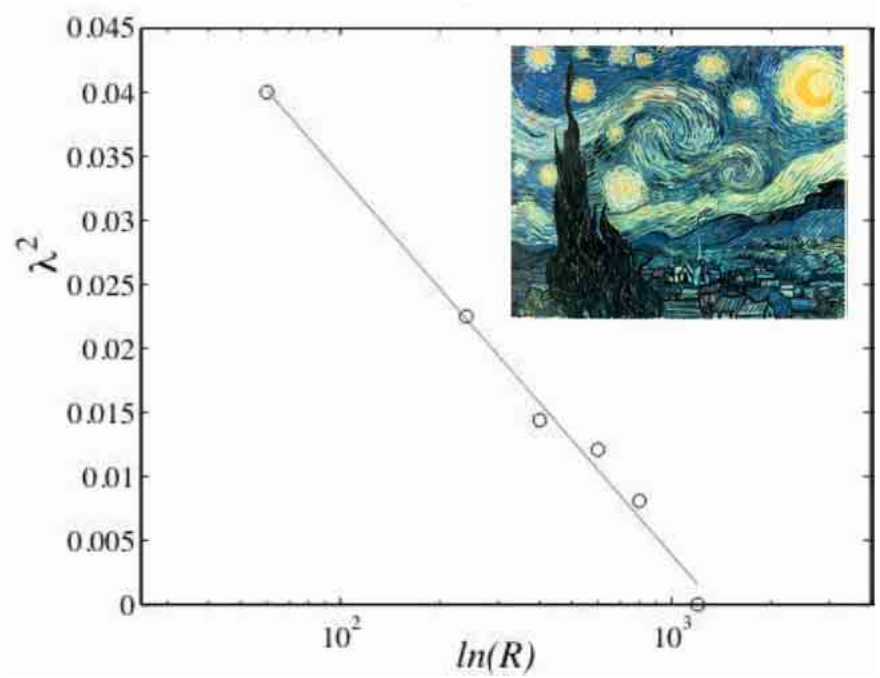
4. **Conclusions**

The Starry Night by Vincent van Gogh (1889)

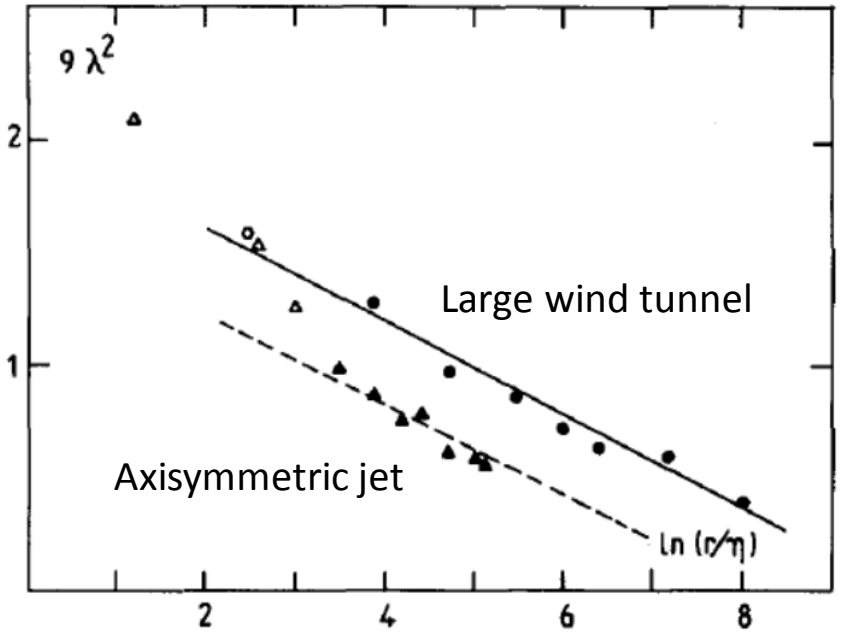
At the Saint-Paul asylum in Saint-Rémy, in southern France

Darkness and Light

Physicists perform the luminance statistics of this picture ….



J.L. Aragon et al. (2006)

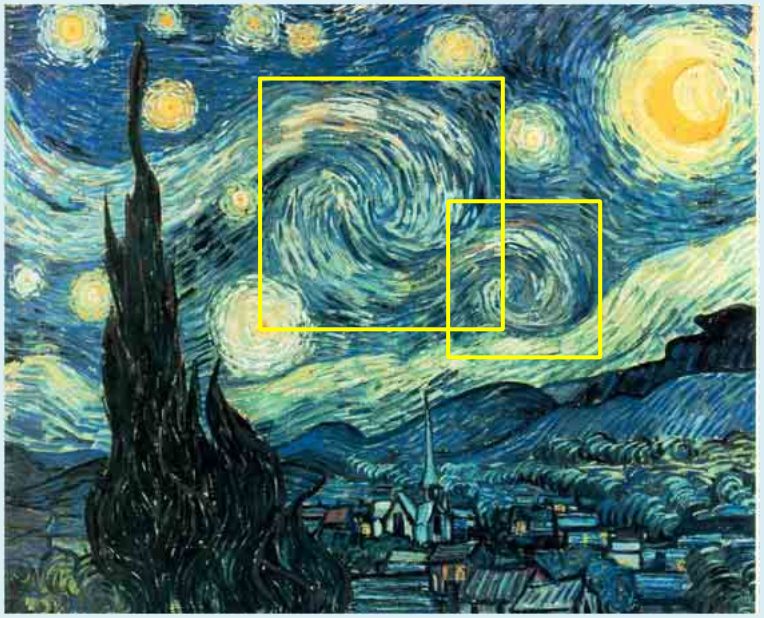


B. Casting et al. (1990)

PDF of luminance fluctuations agrees well with that of velocity fluctuations in turbulent flow predicted by Kolmogorov.

Van Gogh painted turbulence perfectly!

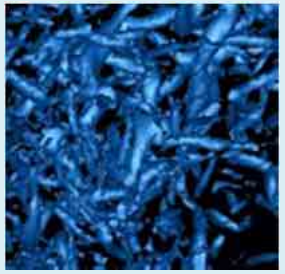
Turbulent Vortices



To the eye, ….

These patterns can be seen as vortices of different sizes.

$= \Sigma$



Large swirl

Fine scale

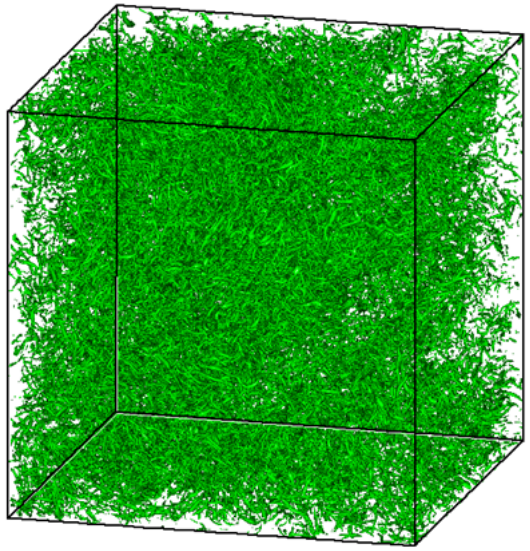
Statistical aspects of turbulent flows as described by K41 theory are the result of such fine scale vortex motions.

Objective

To investigate the relation between hierarchical vortex interactions and energy cascade in turbulent flows

Previous studies

- Reconstruction of flow filed (Vortex method)
- Vortex and shearing motion



Present study

- Comparison of filters (Fourier band-pass filter vs. Component-dependent filter)
- Geometric relations between multi-scale vortices

Forced Homogeneous Isotropic Turbulence

Target turbulent flow filed is calculated by spectral method.

Grid number

Integral scale

Kolmogorov scale

N^3	Re_λ	l_0	λ	η	T_f
128	44	0.93	0.51	0.0039	5.45
256	64	0.87	0.40	0.0026	3.65
512	267	1.18	0.16	0.0050	1.89
1024*	427	1.30	0.12	0.0029	1.92

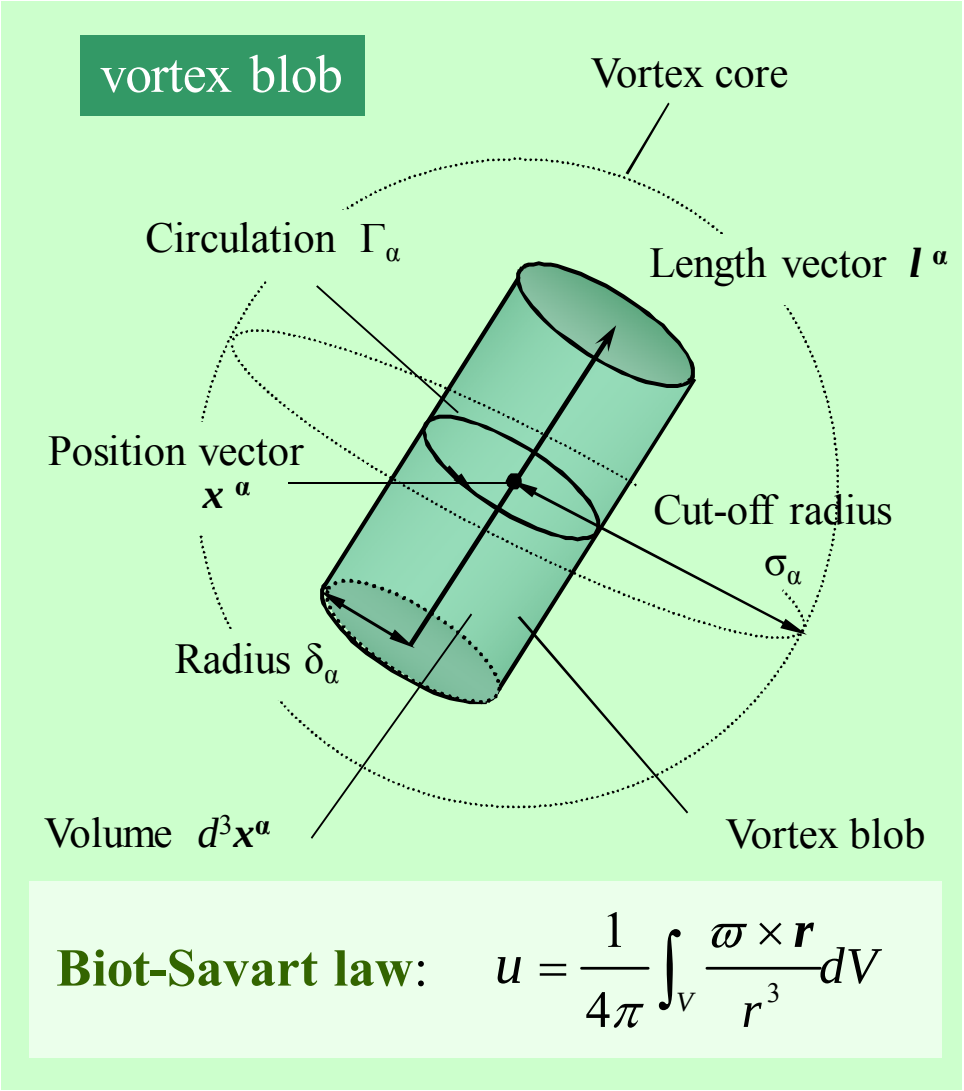
Taylor Reynolds number

Taylor micro-scale

Eddy turn-over time

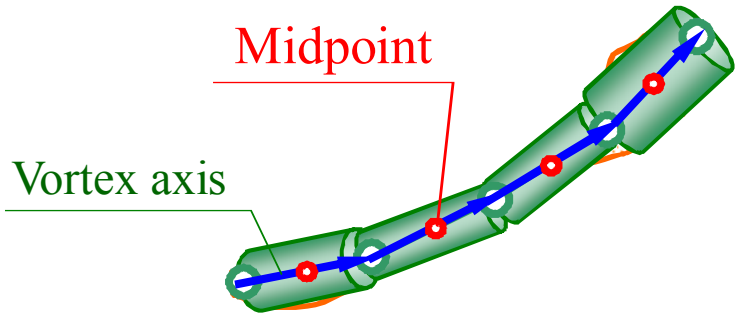
* Johns Hopkins Turbulence Databases: http://turbulence.pha.jhu.edu/

Previous Studies #1 – Reconstruction --



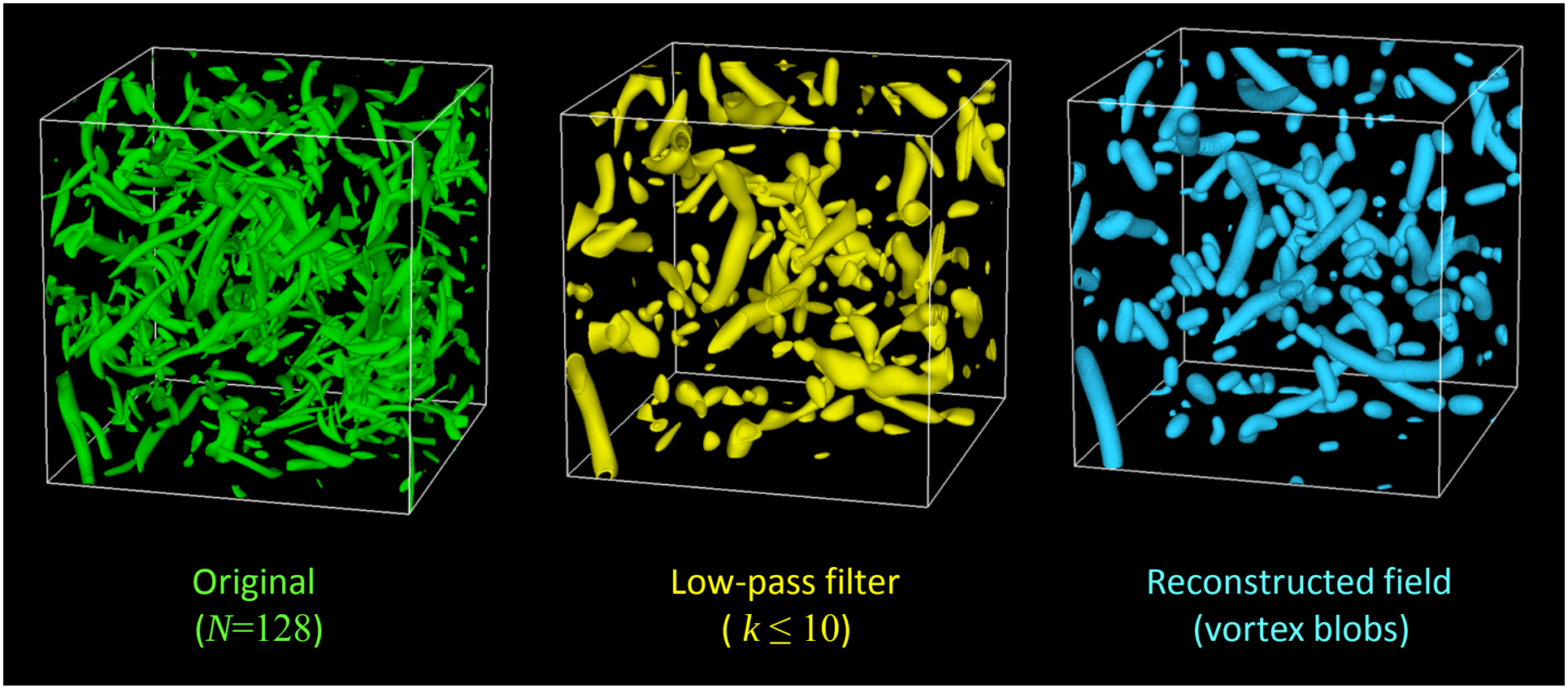
Biot-Savart law: $$u = \frac{1}{4\pi}\int_V \frac{\varpi \times r}{r^3} dV$$

Procedure



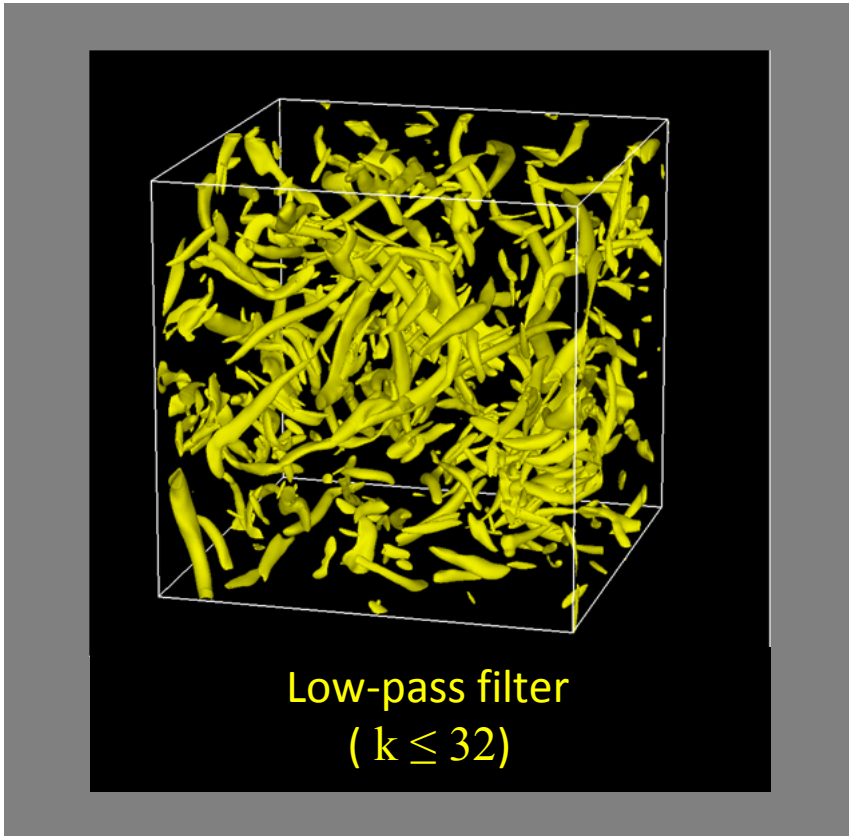
- Each blob is placed at the midpoint.
- Draw the vorticity vector along the vortex axis.
- Blob radius is the average distance from midpoint to point where Q < 0.
- Blob circulation is adjusted to that within the radius, keeping the overlap.
- Cut-off radius is twice of the distance between neighboring blobs.

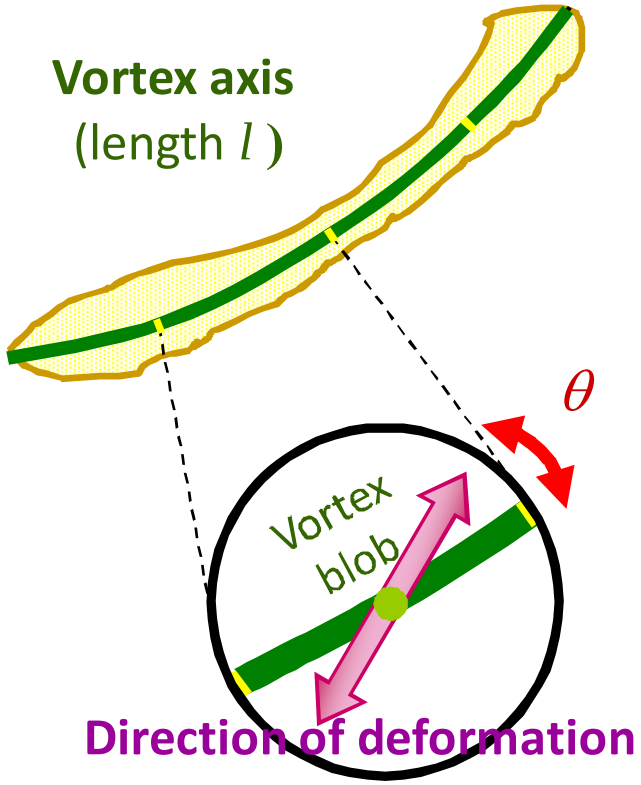
Previous Studies #1 – Reconstruction --



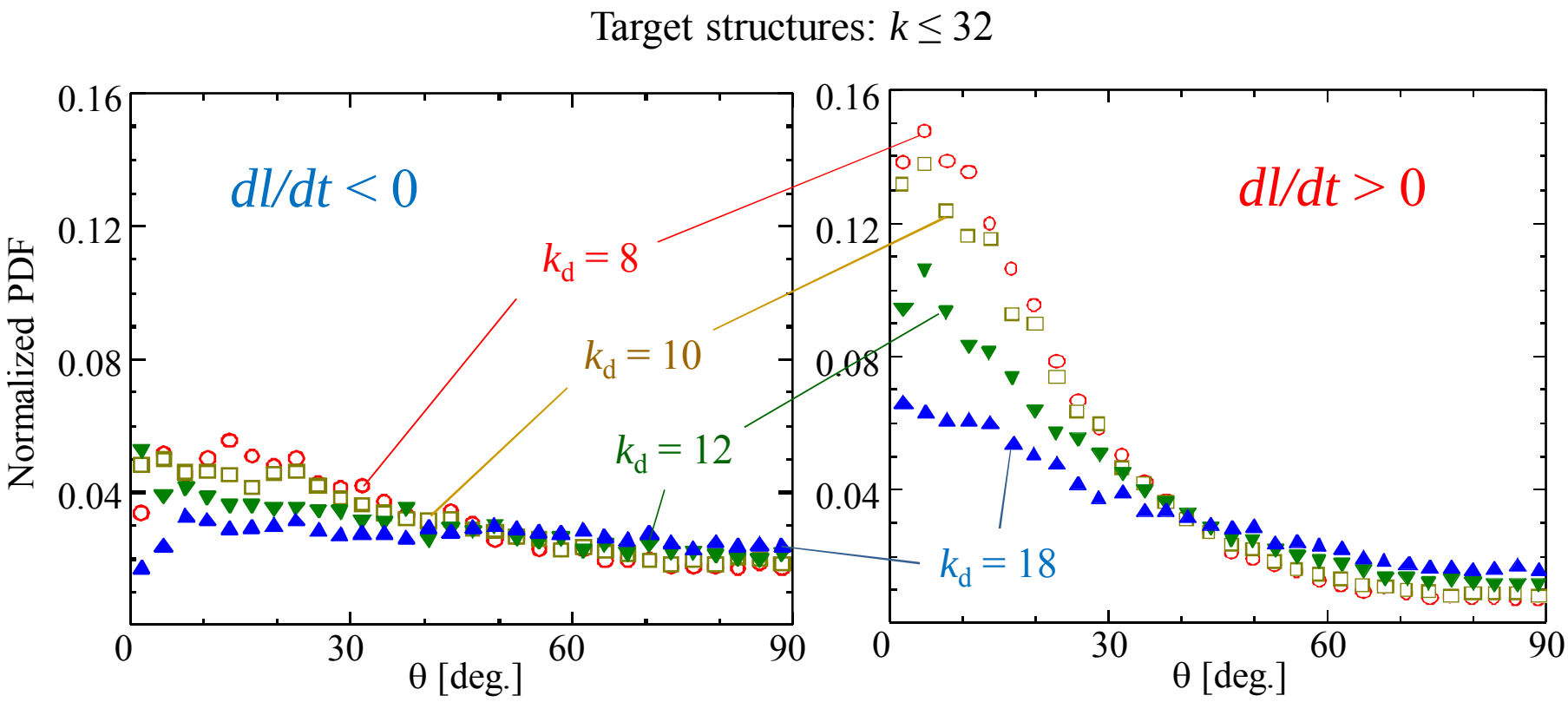
Previous Studies #1 – Vortex and Shearing Motion --

Alignment between **vortex axis** and **deformation vector** defined by the 1st eigenvector of S_{ij} is evaluated focusing on their angle.





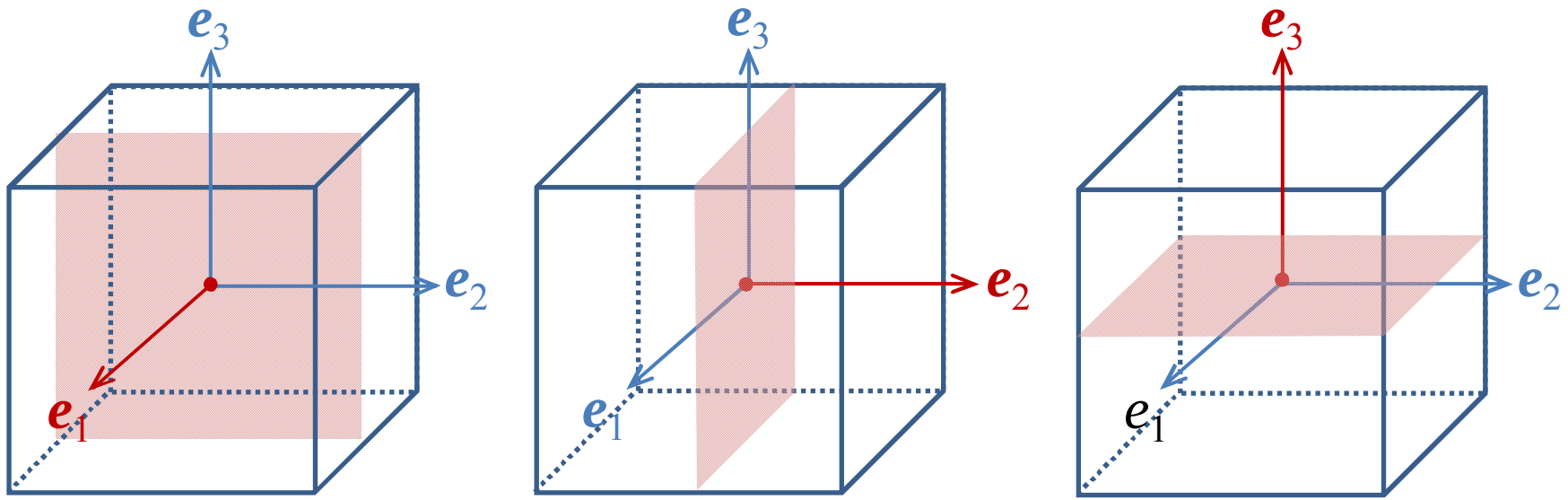
Previous Studies #1 – Vortex and Shearing Motion --



- Vortices not stretching have no relation with the deformation field.
- Stretching vortices tend to be aligned to the deformation field of larger scale.

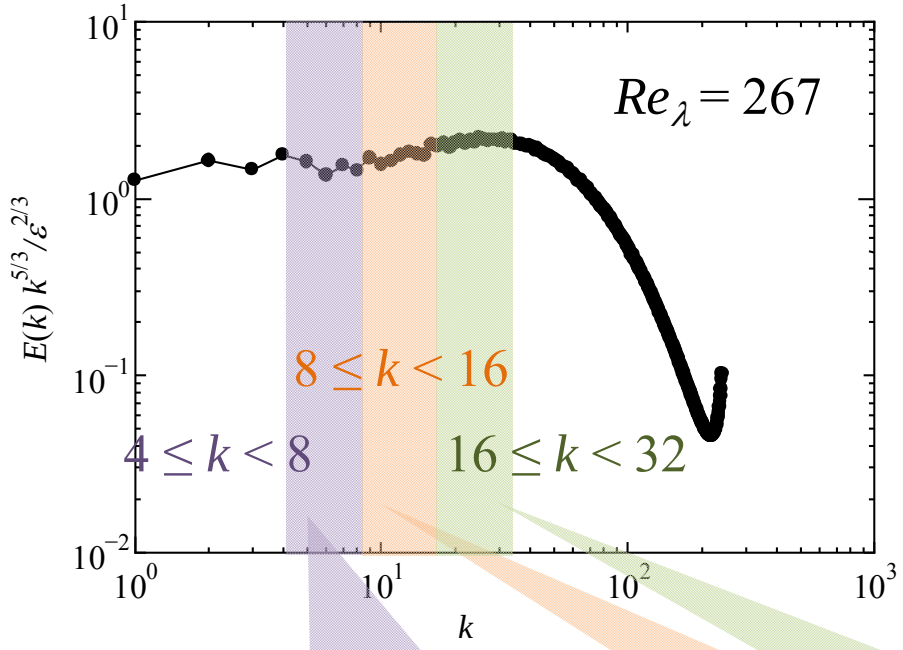
Component-Dependent Filter

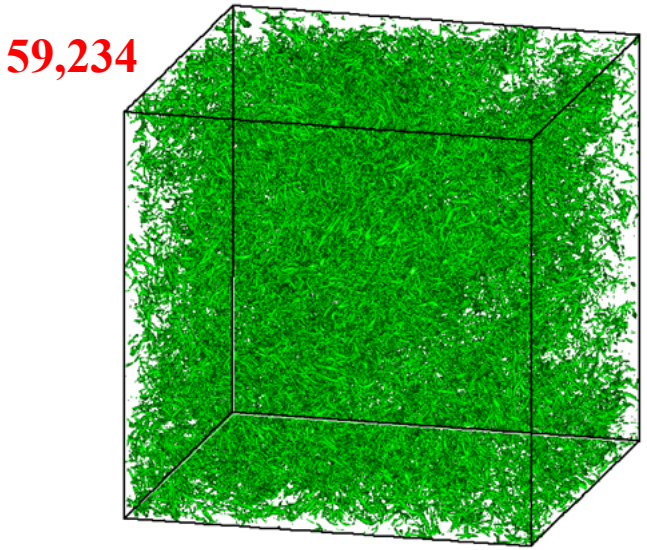
2D Fourier band-pass filter is applied to each plane normal to the direction of basic vectors: $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3 \in \boldsymbol{R}^3$.

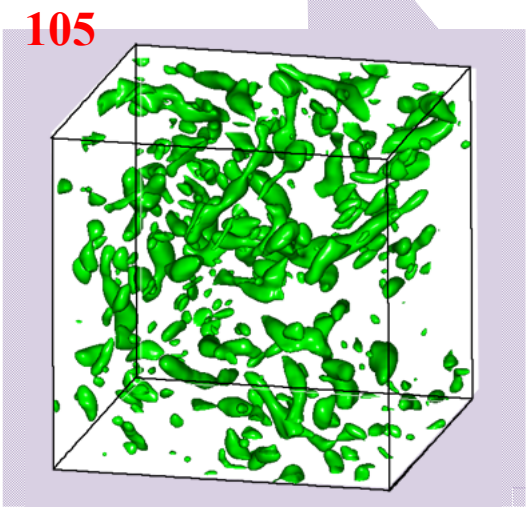


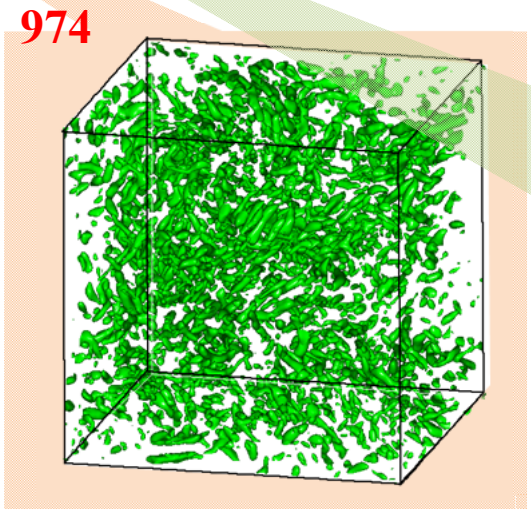
$(\omega_1, \omega_2, \omega_3)$ $(\omega_1, \omega_2, \omega_3)$ $(\omega_1, \omega_2, \omega_3)$

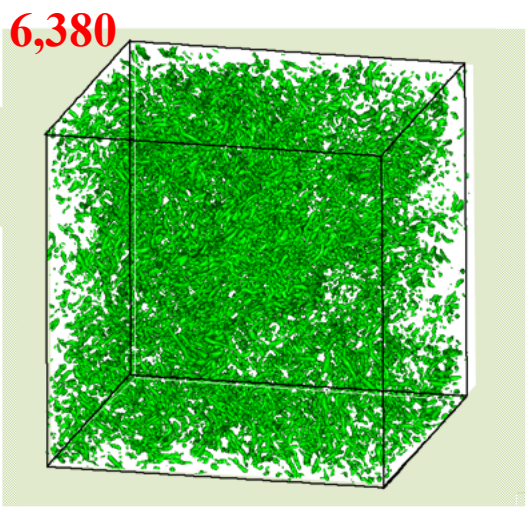
Filtered Structures



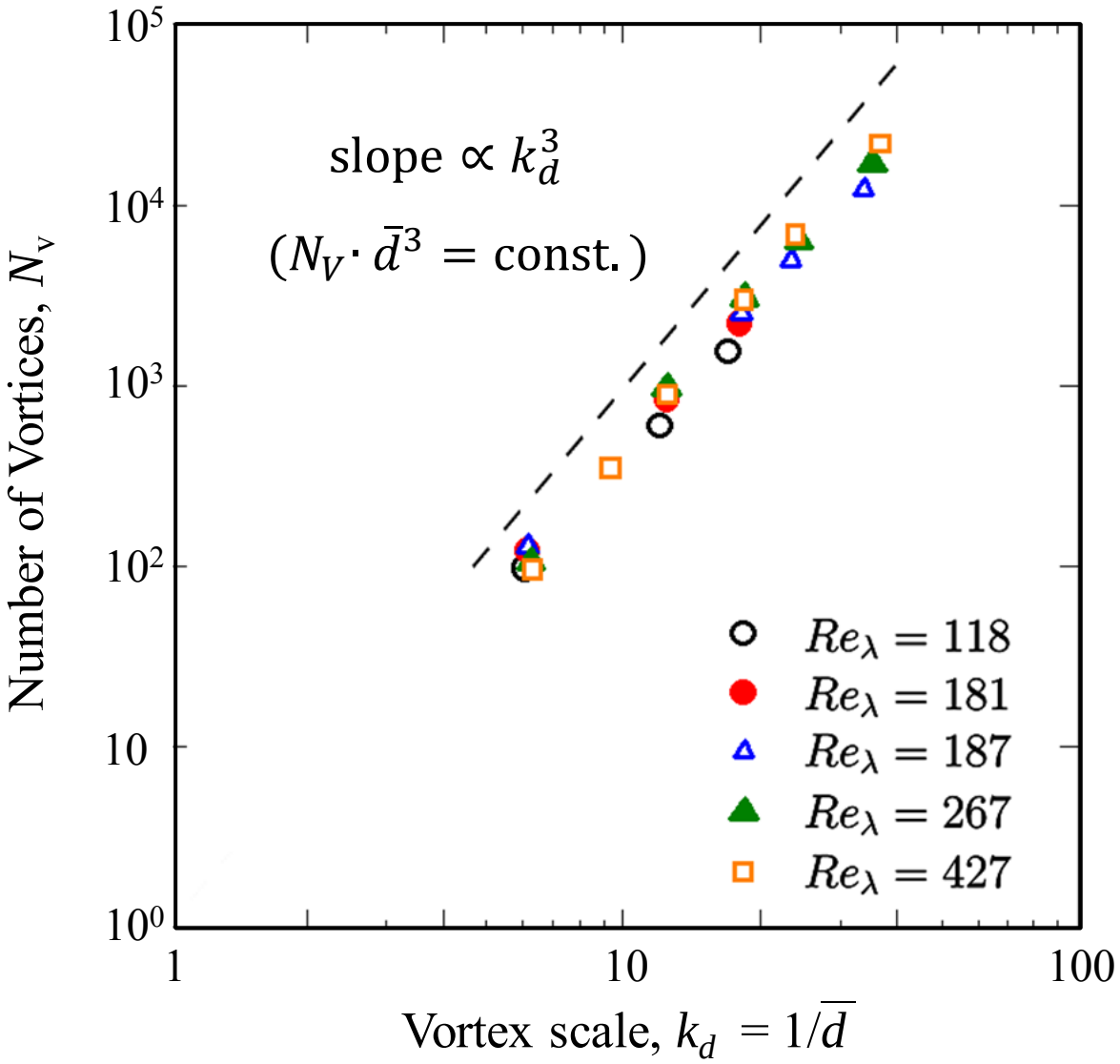




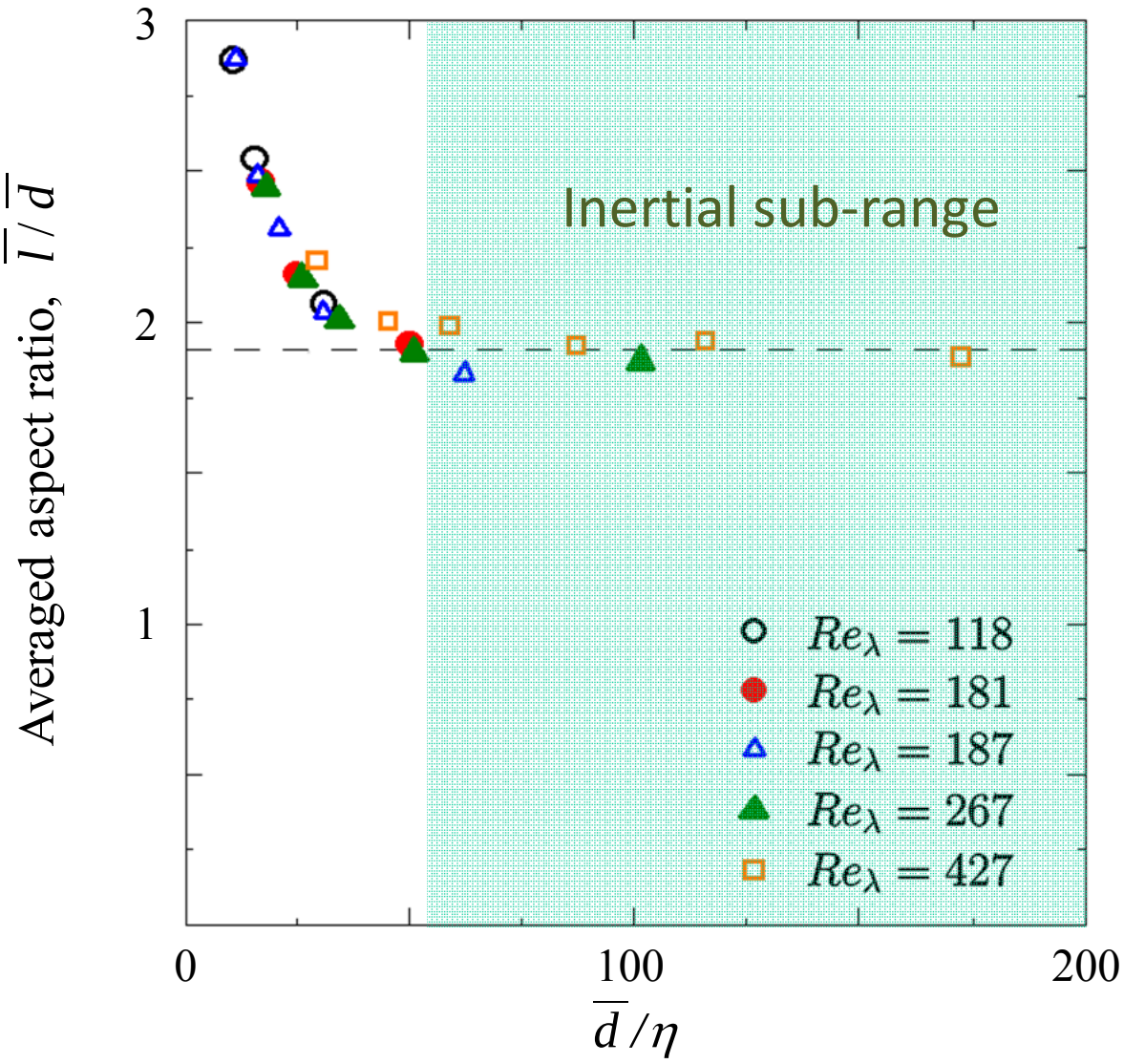




Vortex Scale vs. Number of Vortices

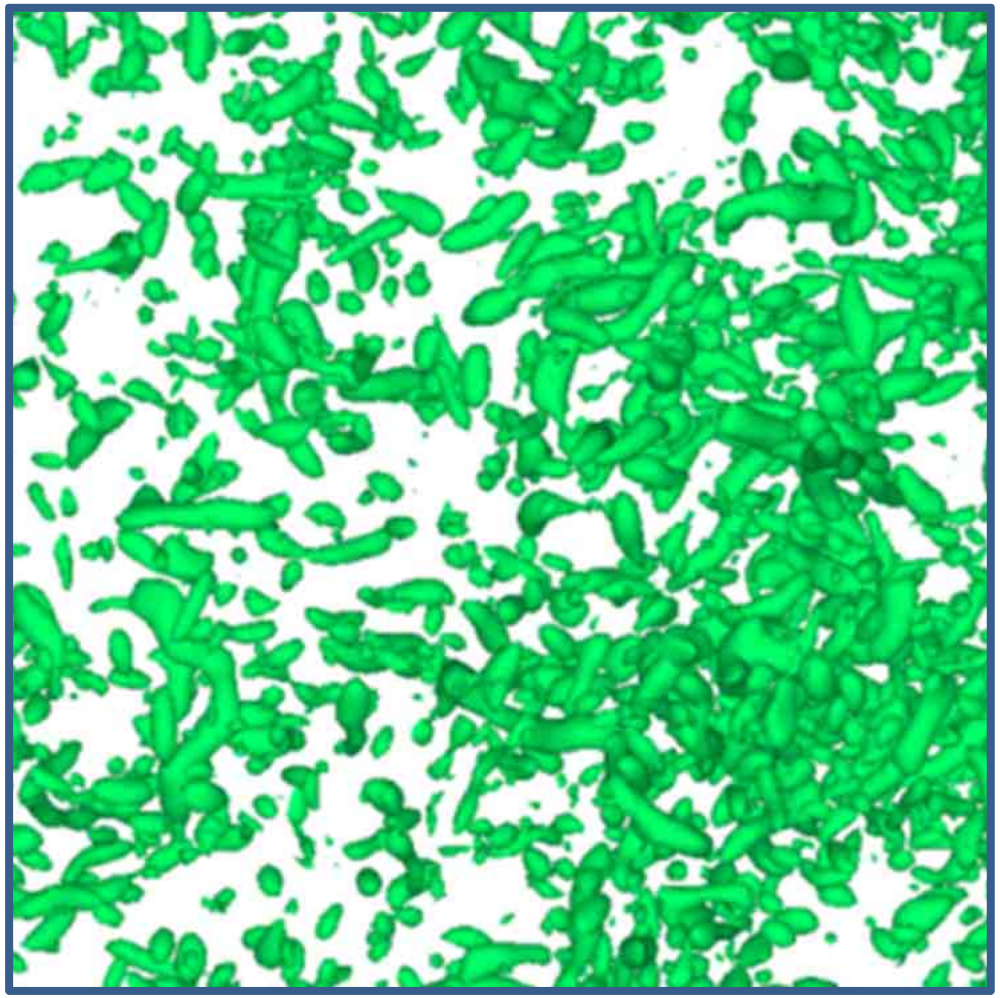


Vortex Scale vs. Aspect Ratio



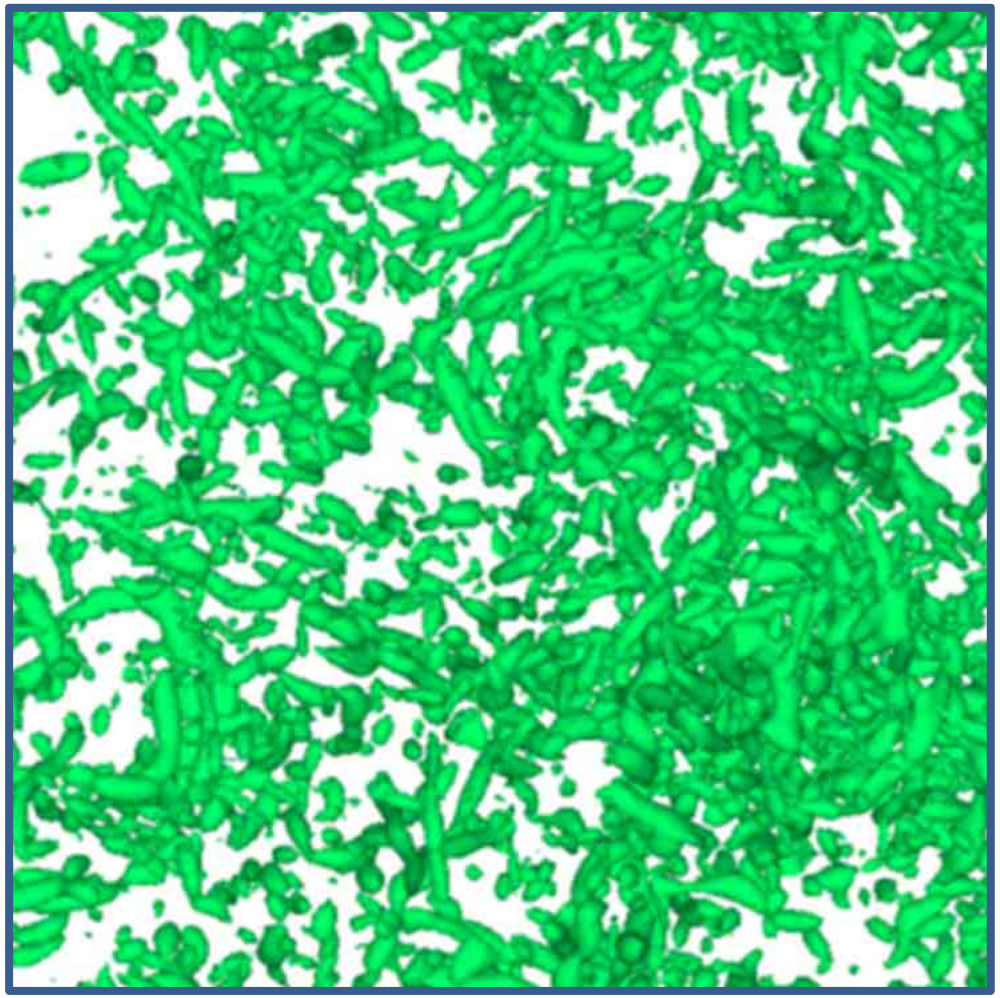
Comparison of Filters #1

Fourier Band-Pass Filter



Isosurfaces of Q, $16 \leq k < 32$

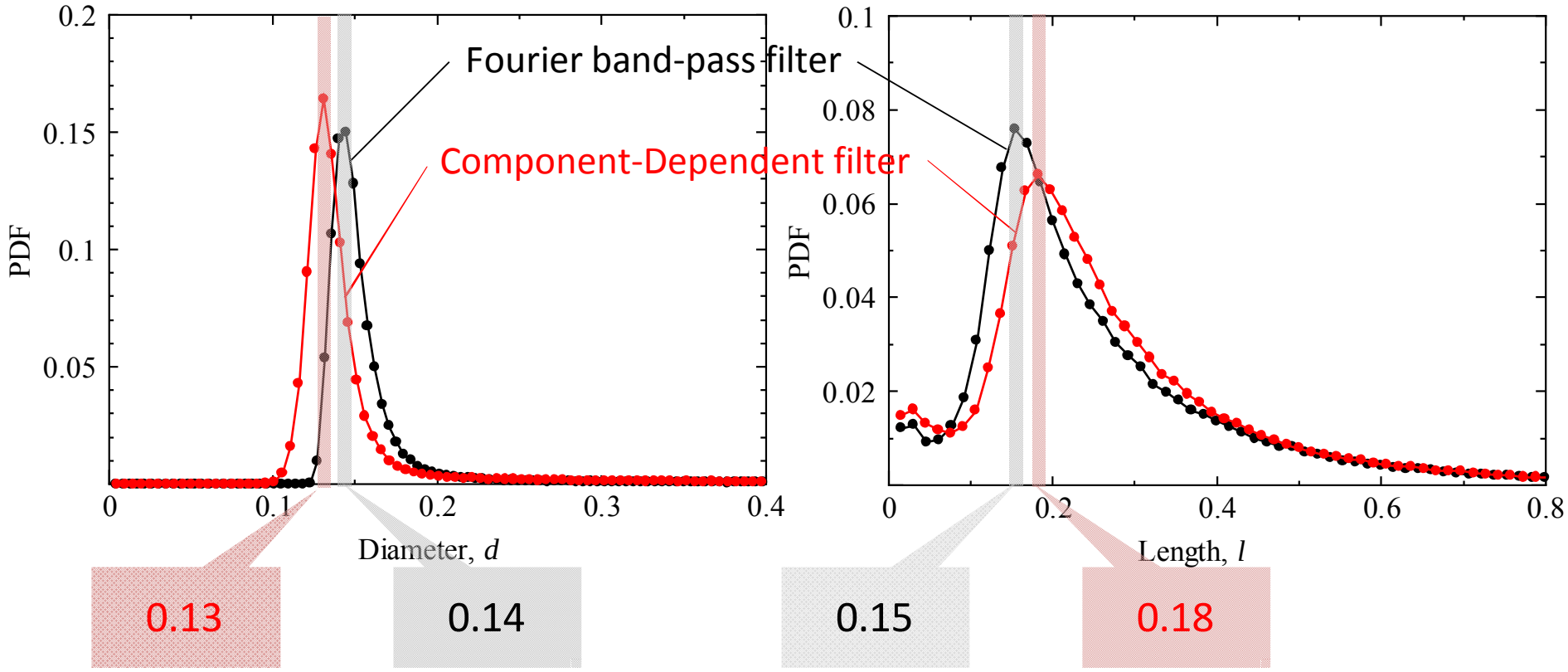
Comparison of Filters #1 Component–Dependent Filter



Isosurfaces of Q, $16 \leq k < 32$

Comparison of Filters #2

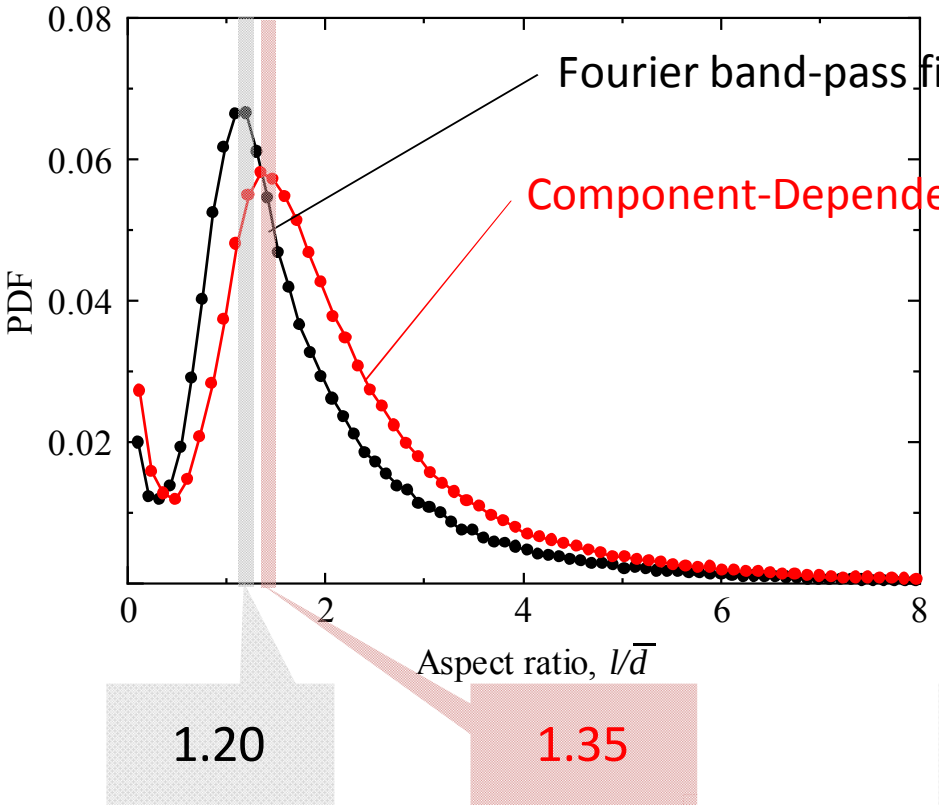
Diameter and length of extracted vortices ($16 \leq k < 32$) are estimated, where tiny vortices are excluded as noise.



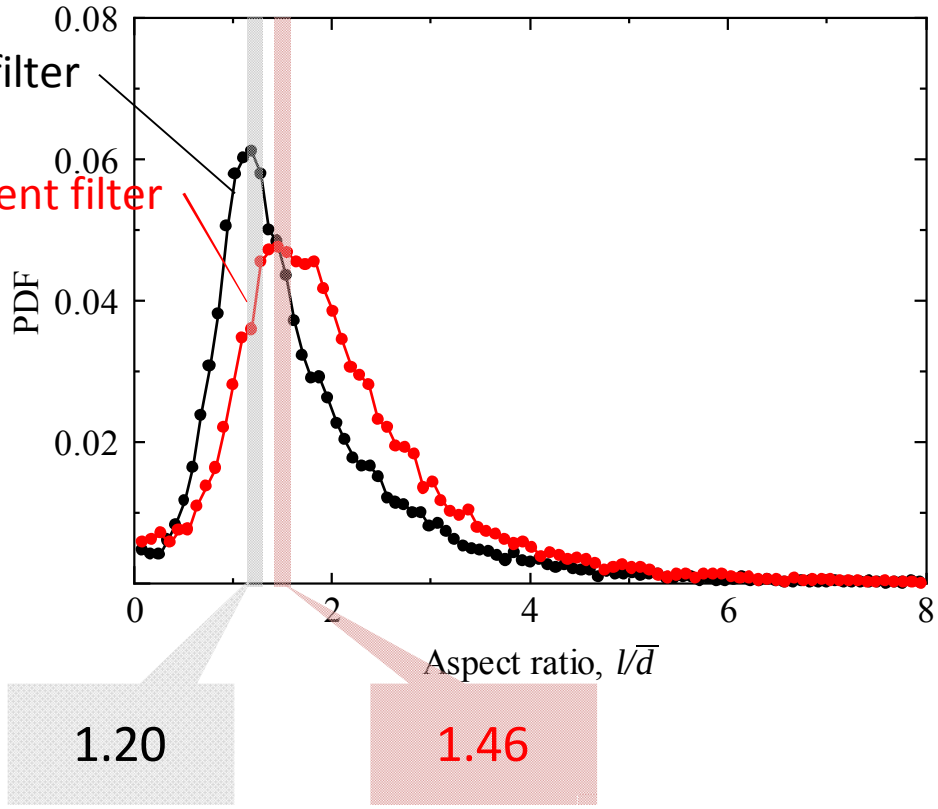
Comparison of Filters #2

Present filter captures smaller and longer vortical structures.

$16 \leq k < 32$



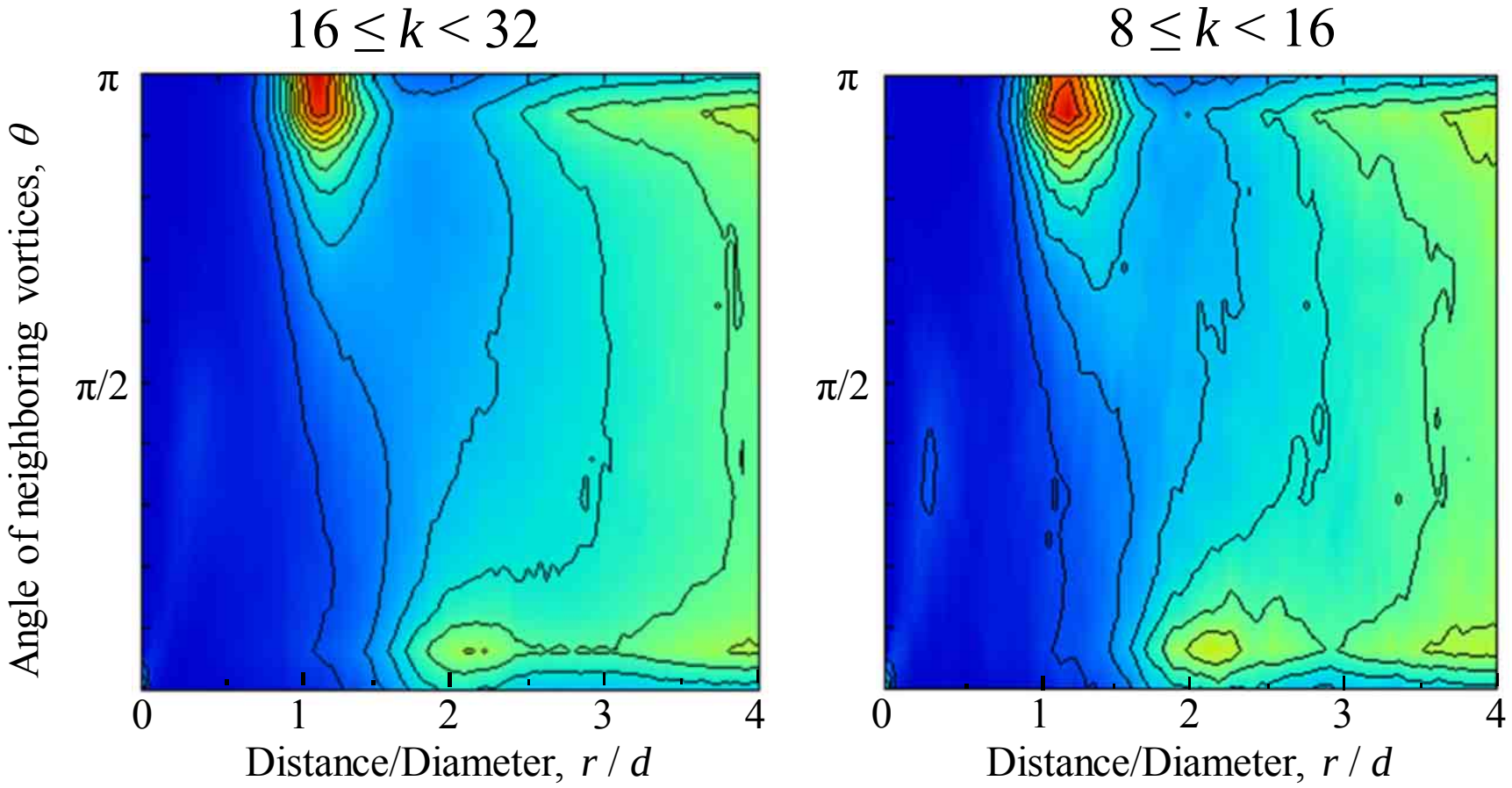
$8 \leq k < 16$



Geometric Relation of Neighboring Vortices

Neighboring vortices tend to be aligned in anti-parallel direction,

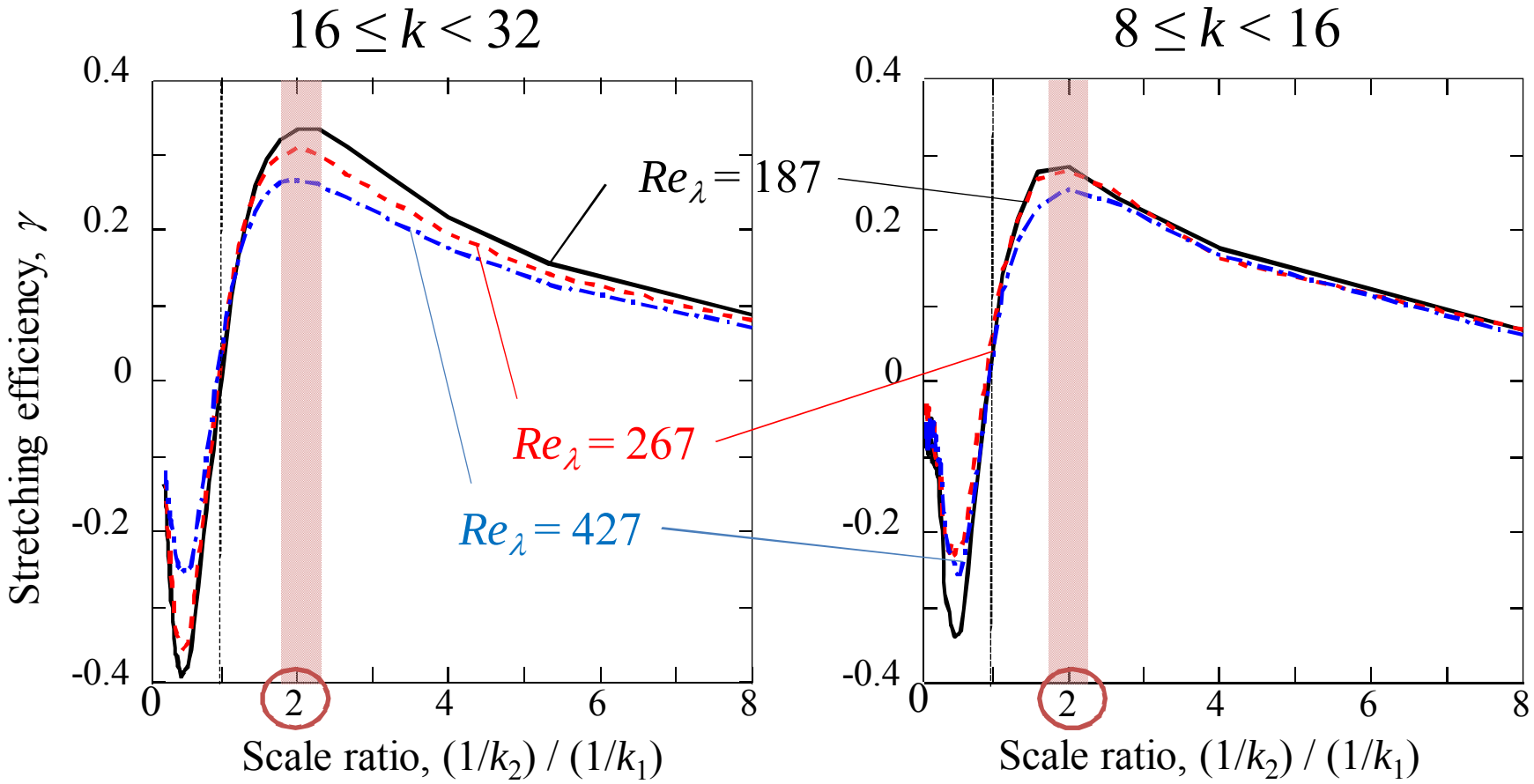
1*d* to 1.5*d* away from each other.



Joint PDF of the angle of neighboring vortices and their distance

Vortex Stretching

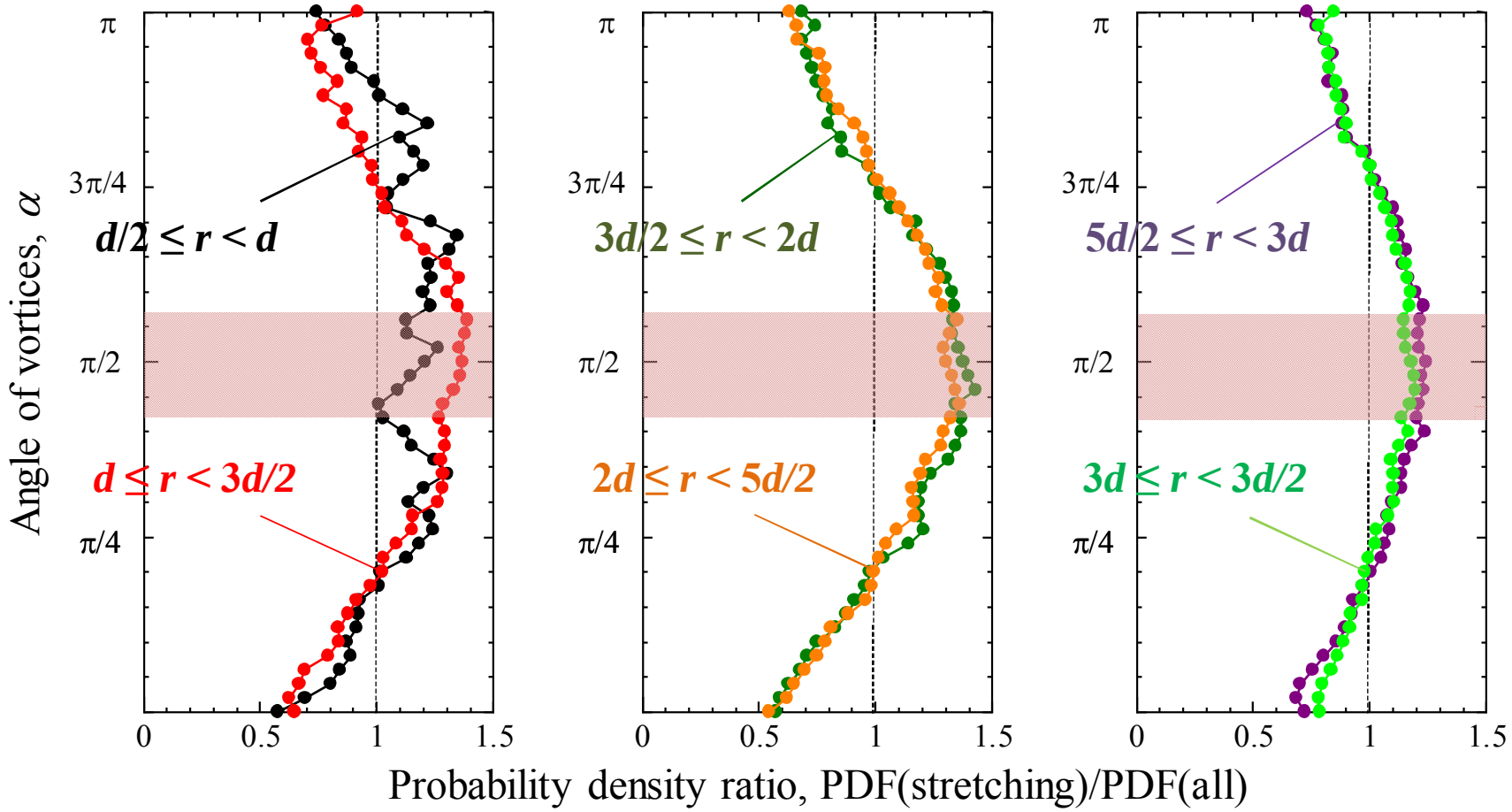
Vortices are likely to be stretched by larger vortices, especially twice as large as themselves.



※ Stretching efficiency is defined as $\gamma = d\ \{(\mathrm{d}\Delta l/\mathrm{d}t)/\Delta l/\}/\ u'_{\mathrm{rms}}$.

Geometric Relation to Double-Scaled Vortices

Stretching vortices ($16 \leq k < 32$) and neighboring double-scaled vortices ($8 \leq k < 16$) tend to be orthogonal to each other.



※ α is the angle between stretching vortices and neighboring double-scaled vortices.

Conclusions

Multi-scaled structures were extracted from a turbulent flow using a **Component-Dependent Filter** and their interaction was studied.

- CDF extracts smaller and longer vortices compared with FB-PF.
- **Self-similarity** is found among extracted vortices.
- Turbulent vortices are likely to be stretched by the velocity filed induced by **double-scaled vortices**, which are **orthogonal** to them.