NUMERICAL STUDY OF THERMOACOUSTIC OSCILLATIONS IN A CLOSED TUBE BY TRACING FLUID PARTICLES

Shizuko Adachi[1] **and Katsuya Ishii**[2]

[1] School of Business and Commerce, Tokyo International University, Japan

[2] Information Technology Center, Nagoya University, Japan,

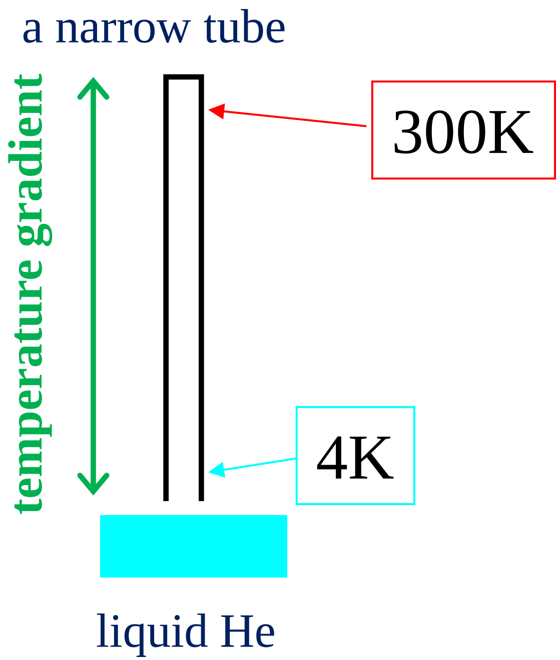
JAPAN-RUSSIA WORKSHOP ON SUPERCOMPUTER MODELING, INSTABILITY AND TURBULENCE IN FLUID DYNAMICS (JR SMIT2015)

Keldysh Institute for Applied Mathematics RAS, Moscow, Russia, March 4-6, 2015

Outline

1 Introduction
1.1 The Taconis oscillation

The Taconis oscillation: spontaneous thermoacoustic oscillations



Experimental studies

Taconis *et a*l. (*Physica*, 1949)
Yazaki *et a*l. (*J.Low.Temp.Phys*., 1980)
Yazaki *et al*. (*PRL*, 1987)

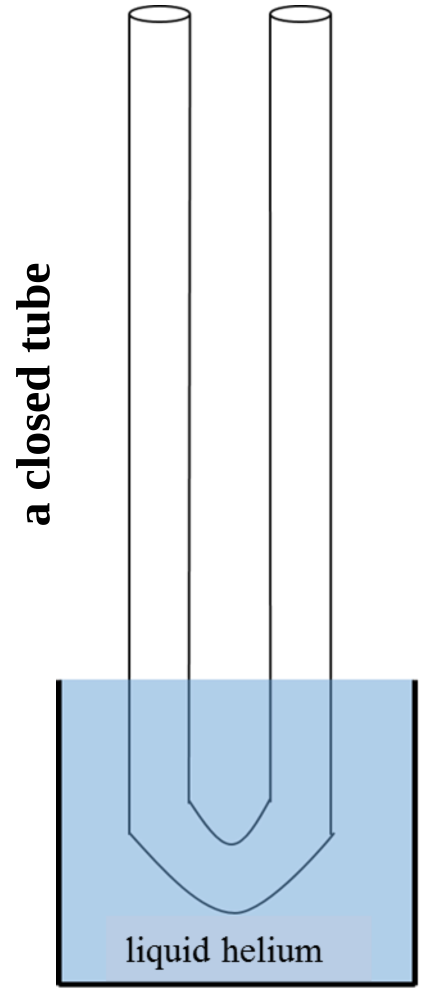
Theoretical studies

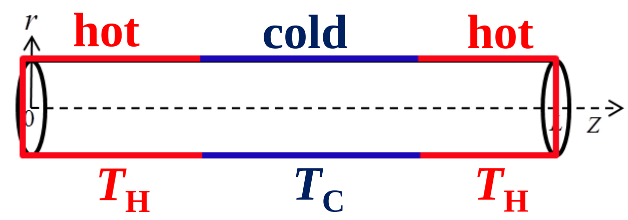
Kramers (*Physica*, 1949)
Rott (*Z.Angew.Math.Phys*.,1969,1973)
Sugimoto *et al*. (*Phys. Fluids*, 2007-2008)
Shimizu and Sugimoto (*J. Appl. Phys*., 2010)

the energy conversion: heat ↔ acoustic energy
→ thermoacoustic engines
← refrigerators

1.2 Experimental study

experiments by Yazaki *et al.*





$$\xi = \frac{\textbf{length of the hot part}}{\textbf{length of the cold part}}$$

They observed
a standing wave
for different ξ's and
different temperature ratios T_H / T_C.

T.Yazaki, A. Tominaga and Y. Narahara, J. Low Temp. Phys. **41**, 45(1980)

experiments by Yazaki *et al.*

hot/cold part length ratio : $\boxed{\xi = 0.3}$

horizontal axis: radius/ thermal boundary layer thickness $R = r/\sqrt{\alpha_C l/a_C}$

vertical axis: temperature ratio T_H / T_C

critical temperature ratio of exp. by Yazaki *et al.*

stability curves : the upper region → oscillations

a standing wave

the lowest frequency mode

the second frequency mode

the lowest freq. mode (antisymmetric)

the second freq. mode (symmetric)

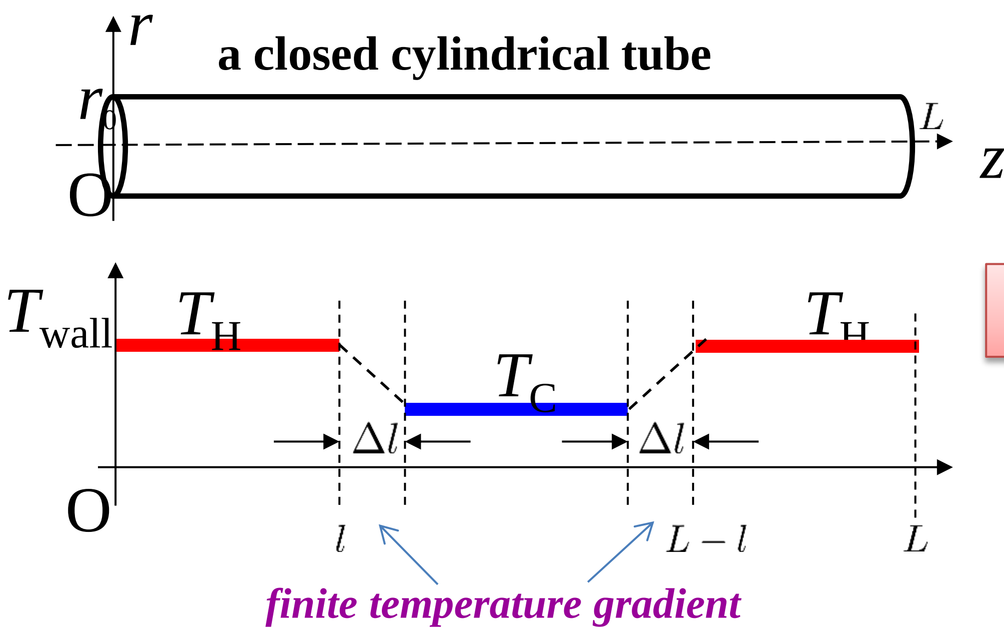
T.Yazaki, S. Takashima and F. Mizutani, Phys. Rev. Lett. **58**, 1108(1987)

1.3 Objectives

1. to present how fluid particles move in different modes of the thermoacoustic oscillations in a closed cylindrical tube
2. to show how thermal quantities change along the particle path
3. to present the work done by fluid particles in different modes

2 The governing equations and Numerical method

2.1 Geometry



T_H=300K T_C=20K ; T_H/T_C=15

the fluid in the tube : gaseous helium

-length ratio of hot part to cold part

$$\xi = \frac{2l}{L-2l}$$

the length ratio $0.2 \le \xi \le 5.0$

-tube length
L=0.28m

-tube radius
r_0=0.76mm

-temp. gradient
Δl=7.5mm

$r_0/L=2.7\times 10^{-3}$

the tube : very narrow

the flow : axisymmetric

2.2 Basic equations — Axisymmetric compressible Navier-Stokes eq.

$$\frac{\partial \boldsymbol{Q}}{\partial t}+\frac{\partial \boldsymbol{E}}{\partial r}+\frac{\partial \boldsymbol{F}}{\partial z}=\frac{1}{\mathrm{Re}}\left(\frac{\partial \boldsymbol{R}}{\partial r}+\frac{\partial \boldsymbol{S}}{\partial z}\right)+\boldsymbol{T}$$

$$\boldsymbol{Q}=r\begin{pmatrix}\rho\\ \rho u_r\\ \rho w\\ e\end{pmatrix}\qquad \boldsymbol{E}=r\begin{pmatrix}\rho u_r\\ \rho u_r{}^2+p\\ \rho u_r w\\ (e+p)u_r\end{pmatrix}\qquad \boldsymbol{F}=r\begin{pmatrix}\rho w\\ \rho u_r w\\ \rho w^2+p\\ (e+p)w\end{pmatrix}$$

$$\boldsymbol{R}=\begin{pmatrix}0\\ \tau_{rr}\\ \tau_{rz}\\ R_4\end{pmatrix}\qquad \boldsymbol{S}=\begin{pmatrix}0\\ \tau_{rz}\\ \tau_{zz}\\ S_4\end{pmatrix}\qquad \boldsymbol{T}=\begin{pmatrix}0\\ p+\frac{1}{\mathrm{Re}}(\tau_{rr}+\tau_{zz})\\ 0\\ 0\end{pmatrix}$$

$$\tau_{rr}=r\mu\left(\frac{4}{3}\frac{\partial u_r}{\partial r}-\frac{2}{3}\frac{u_r}{r}-\frac{2}{3}\frac{\partial w}{\partial z}\right)$$

$$\tau_{rz}=r\mu\left(\frac{\partial u_r}{\partial z}+\frac{\partial w}{\partial r}\right)$$

$$\tau_{zz}=r\mu\left(\frac{4}{3}\frac{\partial w}{\partial z}-\frac{2}{3}\frac{u_r}{r}-\frac{2}{3}\frac{\partial u_r}{\partial r}\right)$$

$$R_4=u_r\tau_{rr}+w\tau_{rz}+r\alpha\frac{\partial a^2}{\partial r}$$

$$S_4=u_r\tau_{rz}+w\tau_{zz}+r\alpha\frac{\partial a^2}{\partial z}$$

$$\alpha=\frac{k}{\Pr(\gamma-1)}$$

$$a^2=\gamma(\gamma-1)\left[\frac{e}{\rho}-\frac{1}{2}(u_r{}^2+w^2)\right]$$

variables : normalized with L (Tube length),
a_0=1004m/s and ρ_0=0.167kg/m^3
(Helium, 100kPa, 300K)
γ=5/3, Pr=0.68
$\mu(T)=\mu_0(T/T_H)^{\beta}$, $k(T)=k_0(T/T_H)^{\beta}$, β=0.647

ρ : density
u_r, w : velocities
e : total energy density
p : pressure

2.3 Numerical method

- The block pentadiagonal matrix scheme

- Time development: 2nd-order accurate three-point backward scheme

$$\frac{3}{2}\Delta Q^n - \frac{1}{2}\Delta Q^{n-1} = \Delta t\left(\frac{\partial Q}{\partial t}\right)^{n+1} + O(\Delta t^3)\,,\quad \Delta Q^n = Q^{n+1} - Q^n$$

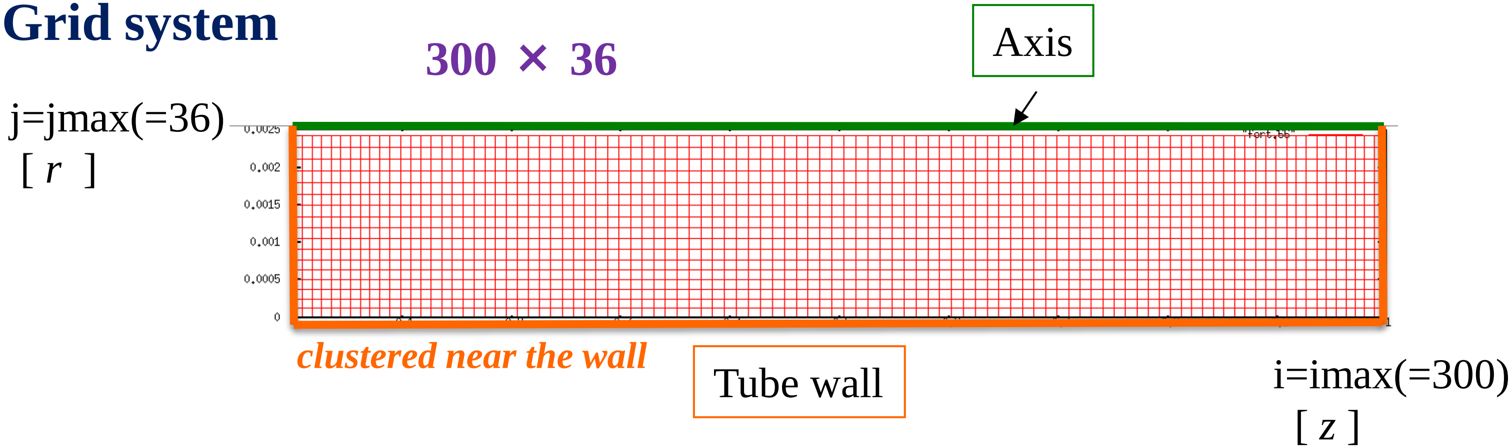
- The approximate factorization method

$$\left\{I - \frac{1}{3}\Delta t\frac{\partial T}{\partial Q} + \frac{2}{3}\Delta t\frac{\partial}{\partial \xi}\frac{\partial}{\partial Q}\left(E - \frac{1}{Re}R_\xi\right)^n\right\}\left\{I - \frac{1}{3}\Delta t\frac{\partial T}{\partial Q} + \frac{2}{3}\Delta t\frac{\partial}{\partial \eta}\frac{\partial}{\partial Q}\left(F - \frac{1}{Re}S_\eta\right)^n\right\}\Delta Q^n$$

$$= -\frac{2}{3}\Delta t\left(\frac{\partial}{\partial \xi}M + \frac{\partial}{\partial \eta}N - T\right)^n + \frac{1}{3}\Delta Q^{n-1} + O(\Delta t^2)\,,\ M = E - \frac{1}{Re}R,\ N = F - \frac{1}{Re}S$$

- Convective terms: 4th-order accurate central differencing
- Viscous terms: 2nd-order accurate central differencing
- The explicit 4th-order artificial dissipation

- Grid system



- Boundary conditions

- [on the wall] non-slip and isothermal boundary conditions, and no pressure gradient in the normal direction of the wall

$$u_r = w = 0 \ \text{ at } \ r = r_0 \text{ and } z = 0,1$$

$$\partial p / \partial r = 0 \ \text{ at } \ r = r_0$$

$$\partial p / \partial z = 0 \ \text{ at } \ \mathrm{z} = 0,1$$

$$T = T_{wall} \ \text{ at } \ r = r_0 \text{ and } z = 0,1$$

- [on the axis] symmetric boundary conditions

$$u_r = 0 \ \text{ at } \ r = 0$$

$$\partial \rho / \partial r = \partial w / \partial r = \partial p / \partial r = 0 \ \text{ at } \ r = 0$$

*How to obtain steady states

the initial state: quiescent and uniform at T_H =300K

with p_0 =1.2 × 10^5Pa

*at ξ = 0.2,

cooling of the central region from 300K to 20K with 5000000 steps

*thereafter T_C =20K fixed : T_H / T_C =15

*ξ is changed by changing the positions of finite temperature gradient.

ξ : from 0.2 to 5.0, then from 5.0 to 0.2

with $\Delta\xi$ = 0.02 or 0.1

*At each ξ , we continue time integration until steady state is obtained.

(more than 500 cycles)

3 Results

3.1 Pressure

> pressure amplitude at the tube end v.s. length ratio ξ

> temporal evolution of the spatial distribution of the pressure

3.2 Temporal evolution of the flow field

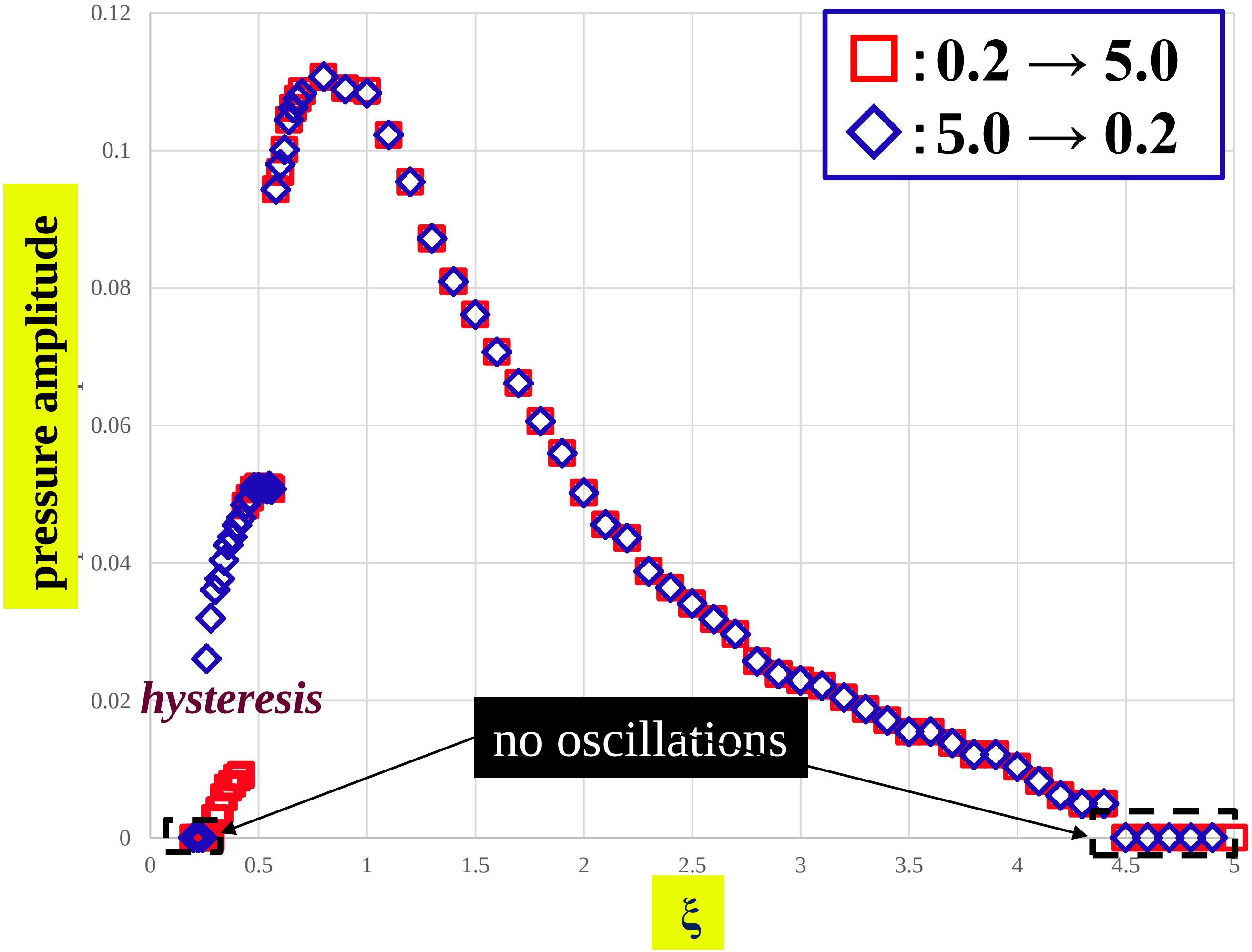
> temperature, axial velocity

3.3 Tracing of fluid particles

> paths of fluid particles

> work done by a fluid particle

pressure amplitude at the tube end v.s. length ratio ξ



measured at
$z = 2.8 \times 10^{-3} L$

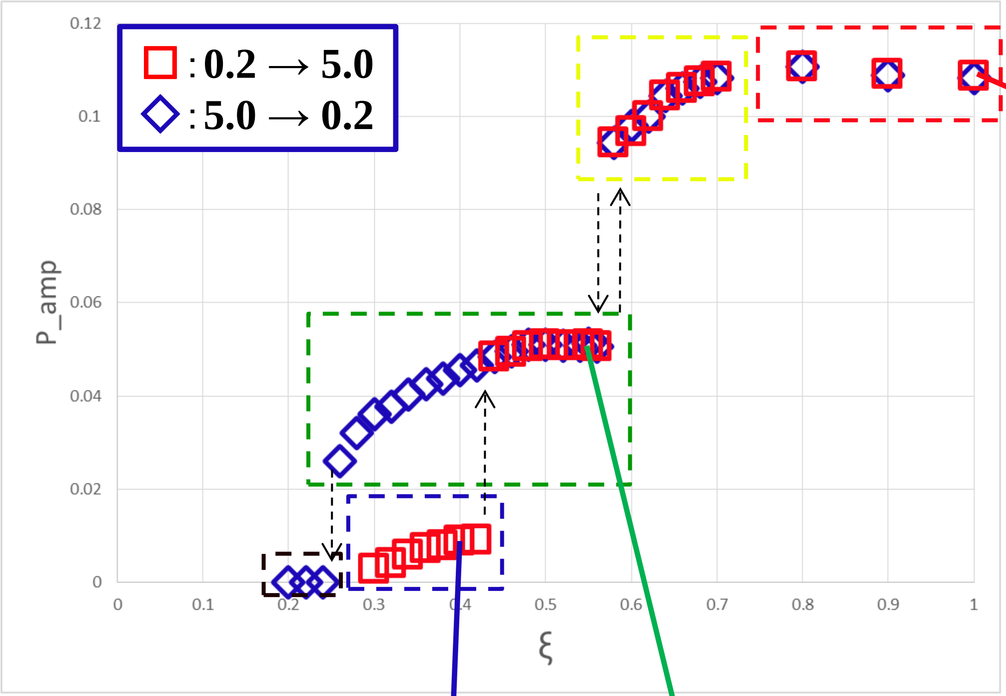
oscillations $\xi = 0.26 - 4.4$

$$\xi = \frac{\textbf{hot part length}}{\textbf{cold part length}}$$

T_H=300K, T_C =20K, θ =15: initial pressure p_0 =1.2 × 10^5Pa

temporal evolution of the spatial distribution of the pressure on the axis

radial dependence: negligible



ξ = 1.0（fundamental mode）
antisymmetric

ξ = 0.4（2nd mode）
symmetric

ξ = 0.5（shock wave mode）

Three modes are observed.

Tracing of fluid particles

> how fluid particles move in a closed tube

> work done by fluid particles during one period
for the second mode ($\xi = 0.4$) and the fundamental mode ($\xi = 1.0$)

$$TdS = dU + pdV$$

the first law of thermodynamics

U : internal energy
T : temperature
S : entropy
p : pressure
ρ : density
V : specific volume

$$\Downarrow$$

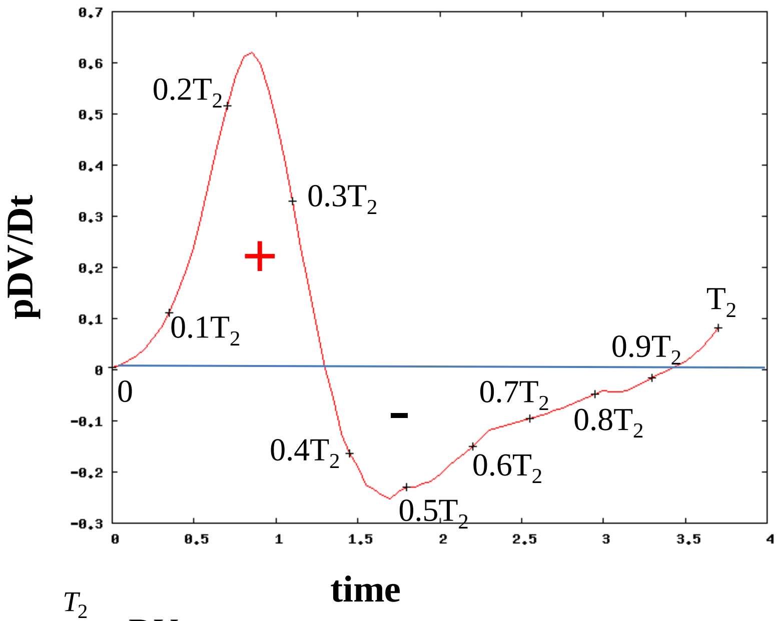
$$\underline{\oint p \frac{DV}{Dt} dt}$$

net work done by a fluid particle during one period
positive : prime mover (heat → work)
negative : heat pump (work → heat)

$\xi = 0.4$ (2nd mode)

- starting point: near the temperature gradient, near the axis

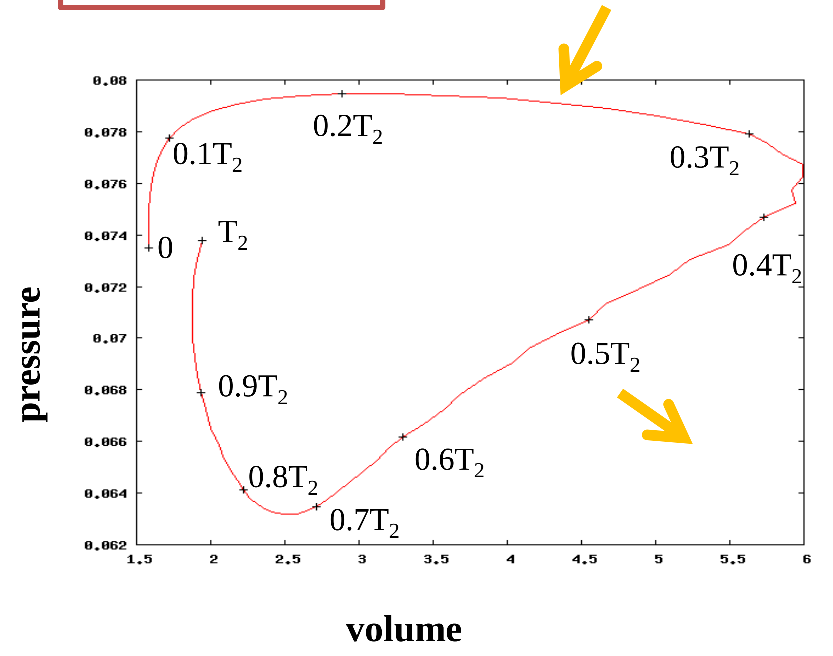
$p\frac{DV}{Dt} - t$ diagram



$$\int_0^{T_2} p\frac{DV}{Dt}dt = \text{positive}$$

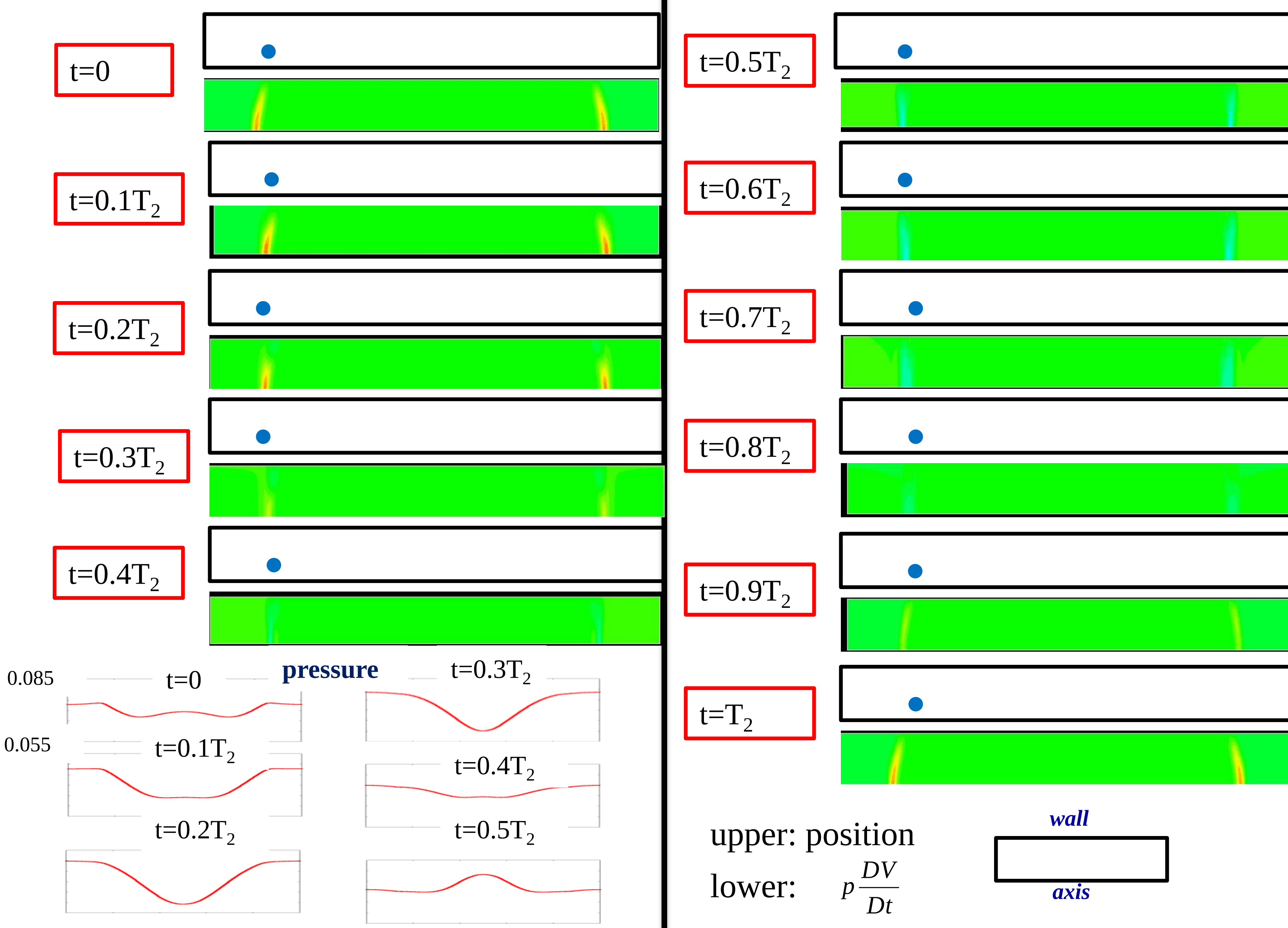
Prime Mover

P-V diagram

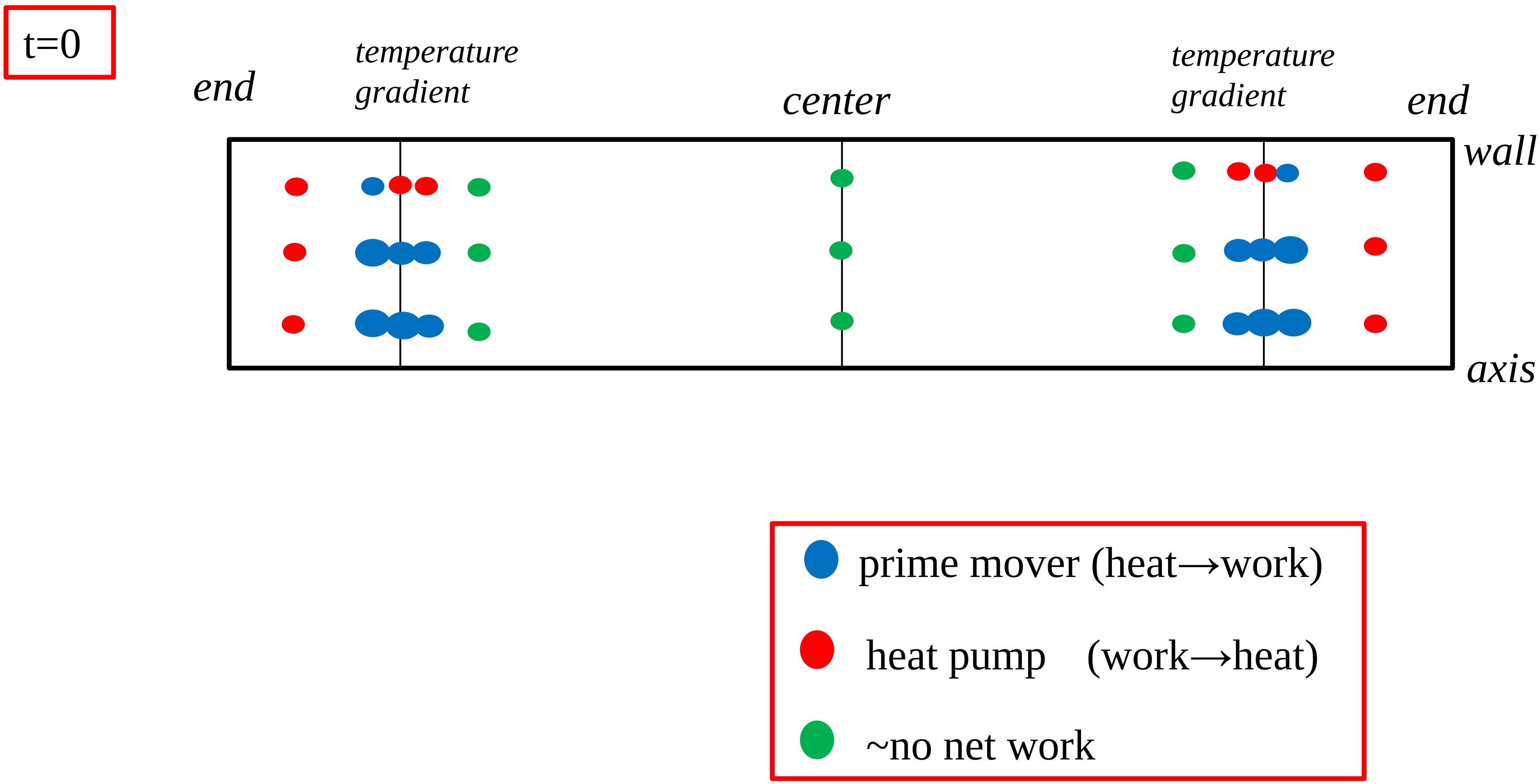


clockwise : positive net work

$\xi = 0.4$ (2nd mode)



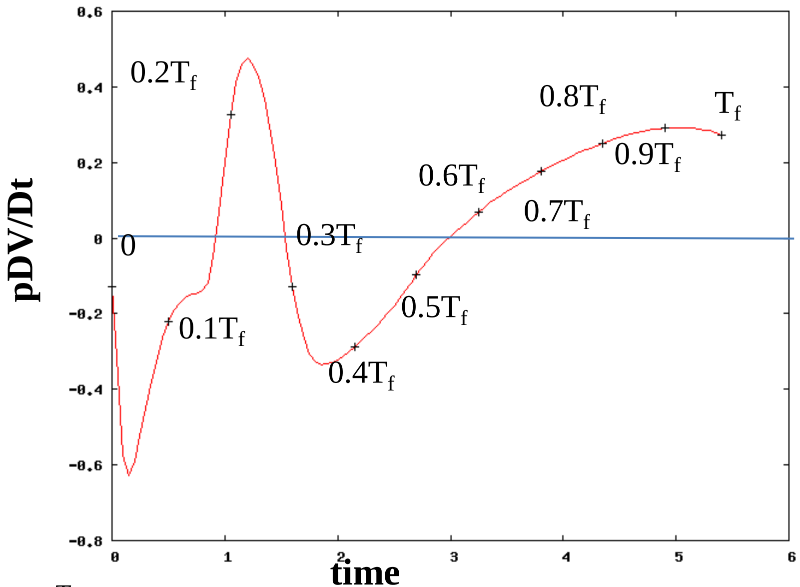
ξ = 0.4（2nd mode）


t=0
end
temperature
gradient
center
temperature
gradient
end
wall
axis
prime mover (heat→work)
heat pump (work→heat)
~no net work

$\xi = 1.0$ (fundamental mode)

$p\dfrac{DV}{Dt} - t$ diagram

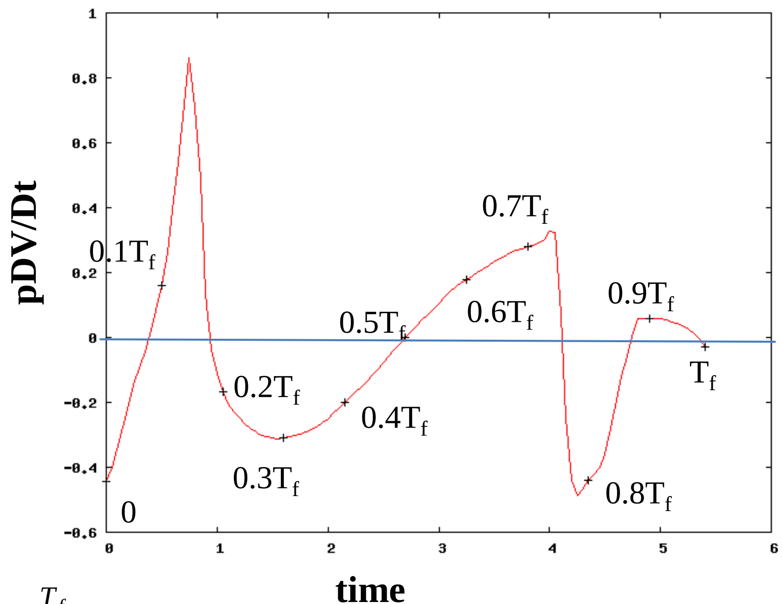
● near the axis



$$\int_0^{T_f} p\frac{DV}{Dt}dt = \text{positive}$$

➡ **Prime Mover**

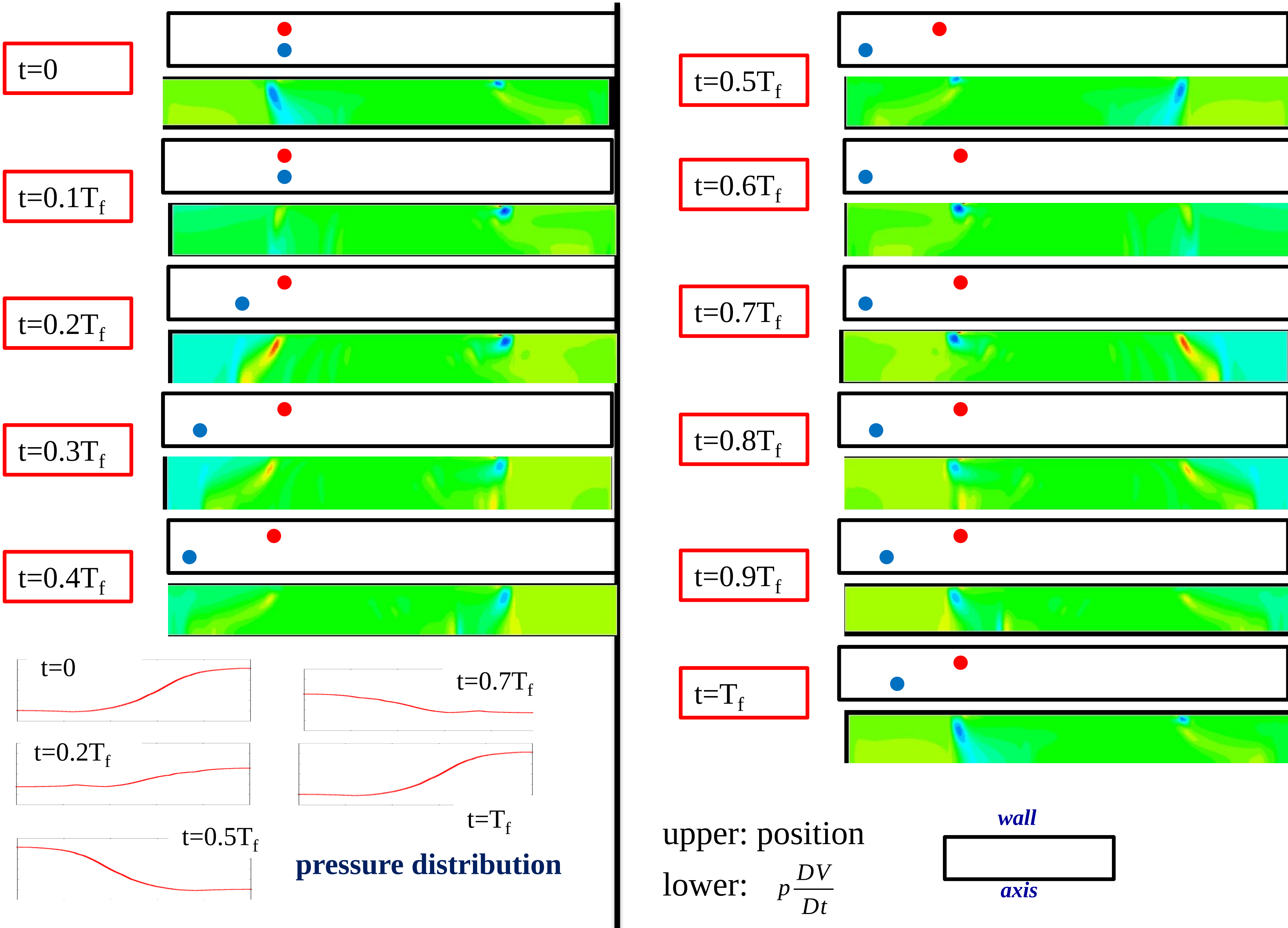
● near the wall



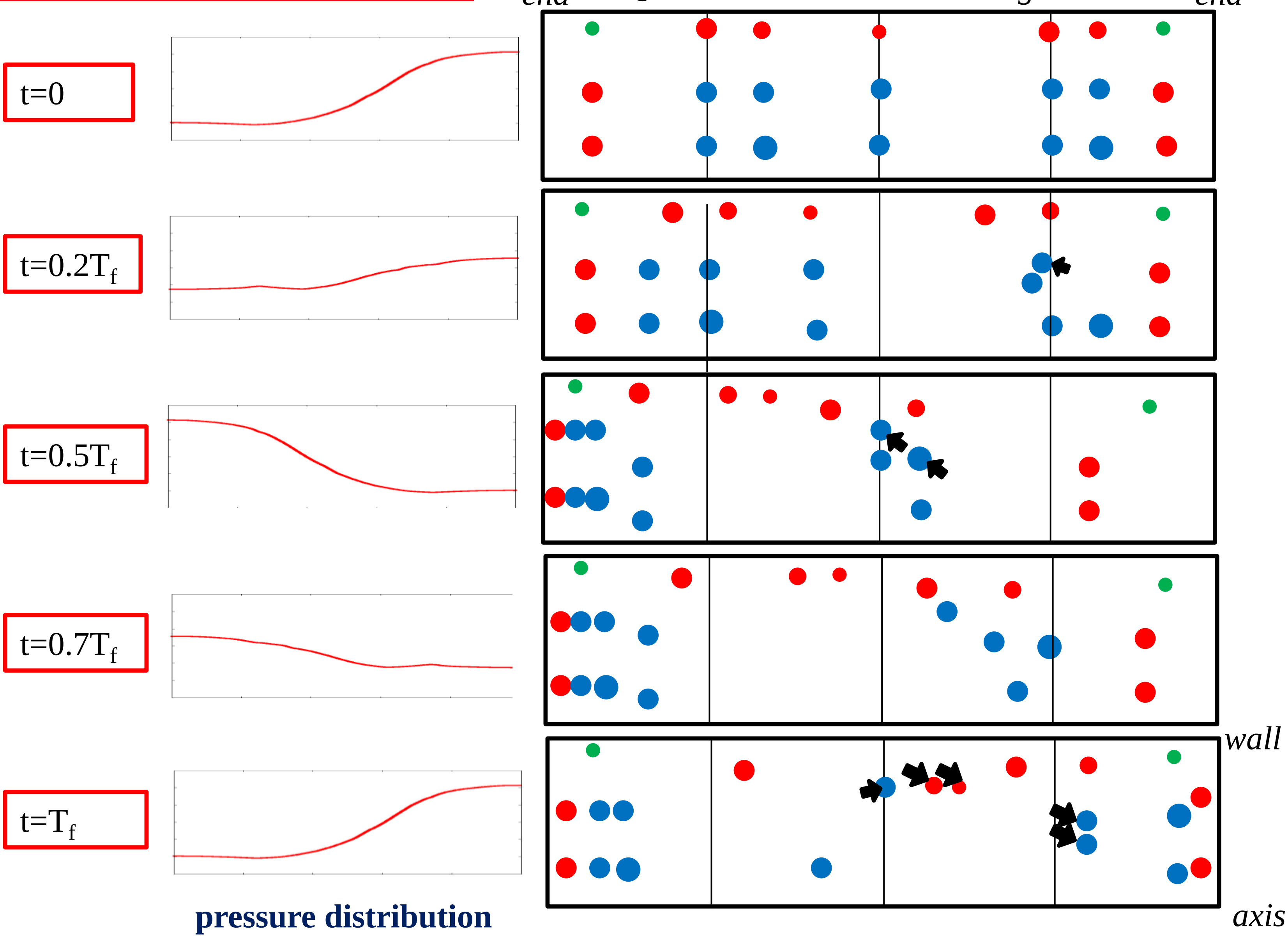
$$\int_0^{T_f} p\frac{DV}{Dt}dt = \text{negative}$$

➡ **Heat Pump**

ξ = 1.0（fundamental mode）



$\xi = 1.0$ (fundamental mode)



prime mover (heat→work)
heat pump (work→heat)
~no net work

5. Summary

- The flow field in a closed cylindrical tube is simulated by solving the axisymmetric compressible Navier-Stokes equations.
- The spontaneous thermoacoustic oscillations of a gas in the tube subject to the temperature gradient is examined.
- the wall temperatures: fixed T_H=300K,T_C=20K: T_H/T_C=15
- the length ratio ξ : changed between 0.2 and 5.0.

1. Tracing fluid particles

   In the second mode, they moves in the vicinity of the starting point.

   In the fundamental mode, their displacement is large.

2. The p-V diagram or the temporal evolution of pDV/Dt shows that they serve as a prime mover or a heat pump.

   The energy conversion is observed.

1. The second mode

- high temperature region(near the tube end) : work → to heat
- low temperature region : ~ no net work
- near the finite temperature gradient :

  - near the wall : work → heat
  - otherwise : heat → work

2. The fundamental mode

- near the tube end: work → heat
- fluid particles moving near the finite temperature gradient:

  during one cycle

  - near the wall : work → heat
  - near the axis : heat → work