JAPAN-RUSSIA WORKSHOP ON SUPERCOMPUTER MODELING, INSTABILITY AND TURBULENCE
IN FLUID DYNAMICS (JR SMIT2015)
March 04, 2015, Keldysh Institute for Applied Mathematics RAS, Moscow, Russia

SIMPLE *A POSTERIORI* LIMITER TOWARDS FLOW SIMULATIONS WITH 64 TIMES HIGHER RESOLUTION

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and
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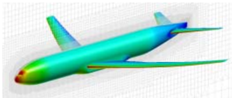
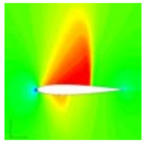
YNU YOKOHAMA National University

Present (Compressible) CFD

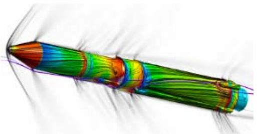
0.01 | 0.1 | 0.7 | 1.0 | 1.2 | 1.5 | 5 → M



Incomp-ressible



Tran-sonic



Hypersonic

Subsonic | **Supersonic**

Low-Speed

- Anomalous Solutions due to Excess Dissipation
- Degraded Convergence

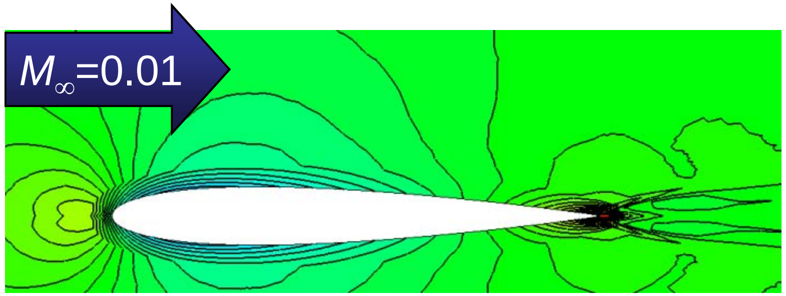
Almost **Matured**

➡ Current interests:

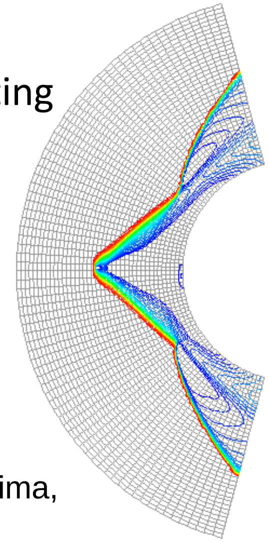
- **Error Reduction**
- Convergence Acceleration

High-Speed

- Shock Anomalies (e.g., Carbuncle)
- Difficulties in Heating Prediction



Kitamura et al., *CiCP*, 2011.

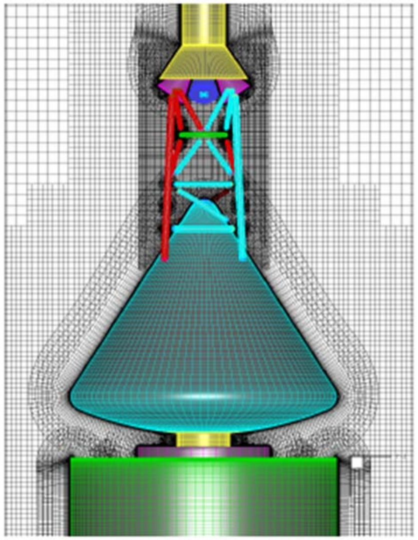


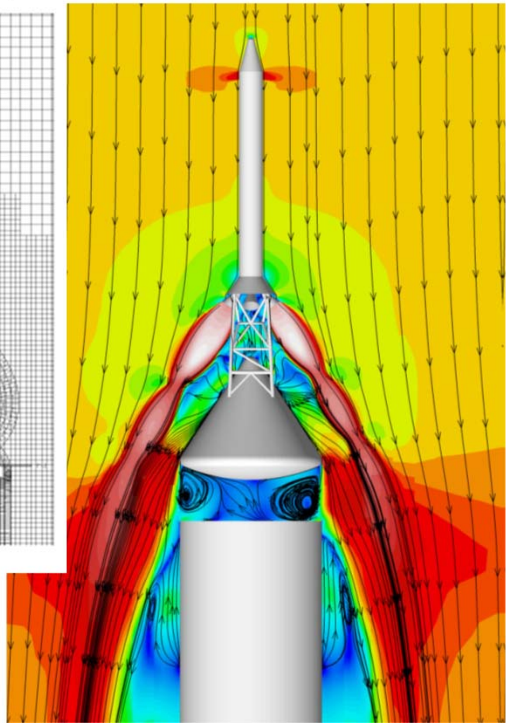
M_∞=8.1

Kitamura & Shima, *JCP*, 2013.

Recent Aerodynamic Simulations

Ares I: Cart3D – Cartesian, Automated (NASA Ames)

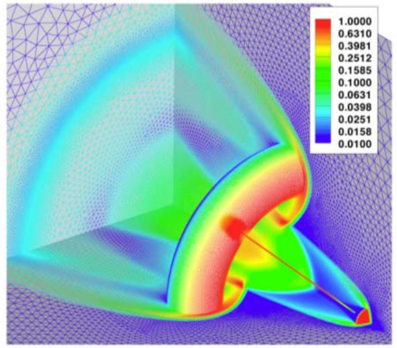




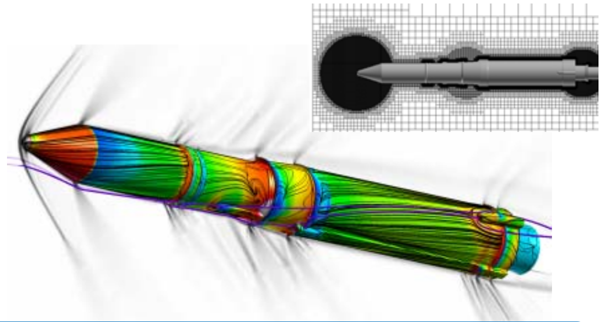
Pandya et al., NAS-06-005, 2006; AIAA 2006-90

Ballute: FUN3D – Tetra, Unstructured (NASA Langley)

Gnoffo et al., AIAA-2006-3771, 2006.



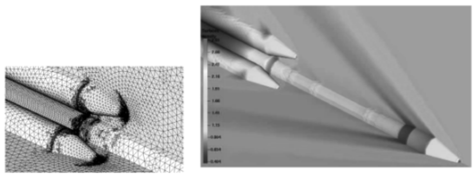
Epsilon Launch Vehicle: LS-GRID/FLOW – Body-Fitted Cartesian (JAXA)



Kitamura et al., *JSR*, 2013.

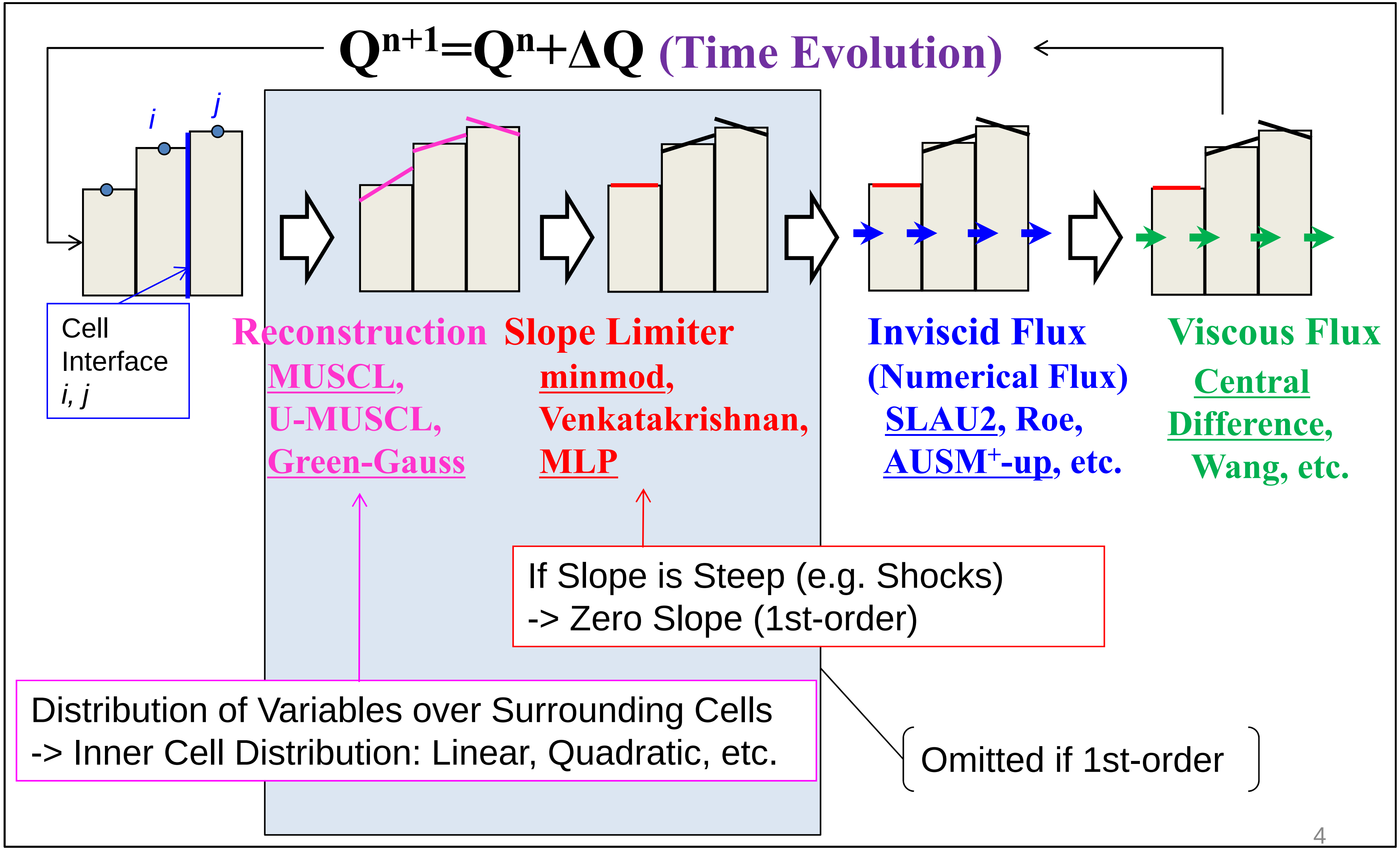
Brazilian Satellite Launch Vehicle: Unstructured

Bigarella et al., *JSR*, 2007.



Unstructured, spatially 2nd-order:
Still in the "mainstream"

Schematic of One Timestep in Finite Volume Method: Spatially 2nd-order (or higher)



Computational Methods

- Governing Eqs.: **2D** compressible **Euler/N-S** Eqs.
- Spatial Discretization: Cell-centered **FVM**
  - Gradient: **MUSCL** (κ= -1 or 1/3: 2nd order) or **Green-Gauss**
  - Slope Limiter: **minmod**, **MLP**, or none
  - Inviscid Term (Numerical Flux): **SLAU2** or **AUSM$^+$-up**
  - Viscous Term: **Central Difference** (2nd order)
- Temporal Evolution: **Runge-Kutta** (2 or 4)

$$q_{i,j} = q_i + \phi \nabla q_i \bullet (\mathbf{r}_{i,j} - \mathbf{r}_i)$$

Slope

Limiter [0,1]

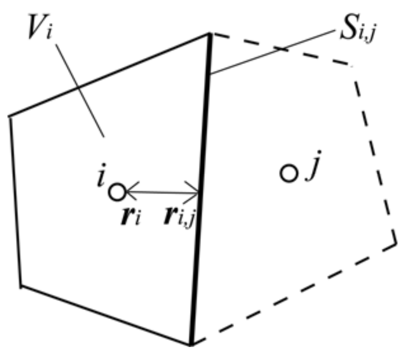
Time Evolution

$$\mathbf{Q}_i^{n+1} = \mathbf{Q}_i^n - \frac{\Delta t_i}{V_i} \sum_j (\mathbf{F}_{i,j} - \mathbf{Fv}_{i,j}) S_{i,j}$$

Variable Vector at (n+1)-th Timestep

Inviscid Flux

Viscous Flux



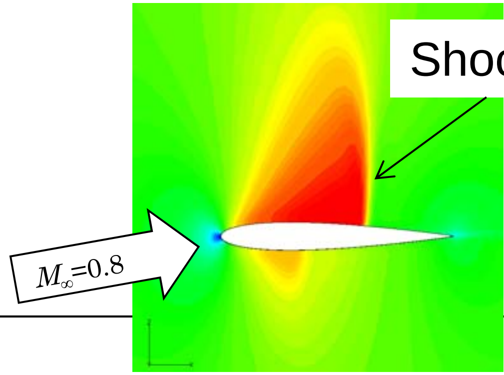
(Conventional, ***a priori***) Limiter

- Spatial Accuracy: usually 2nd-order or higher

➢ At Discontinuities (e.g. Shocks): Over/Undershoot

➢ "Limiter ϕ" lowers the order of accuracy ***a priori***, where the computation is "likely" to get unstable

= Too much Limiting ? (ϕ=0)



Time Evolution

$$\mathbf{Q}_i^{n+1} = \mathbf{Q}_i^{n} - \frac{\Delta t_i}{V_i}\sum_j (\mathbf{F}_{i,j} - \mathbf{Fv}_{i,j}) S_{i,j}$$

Variable Vector at (n+1)-th Timestep

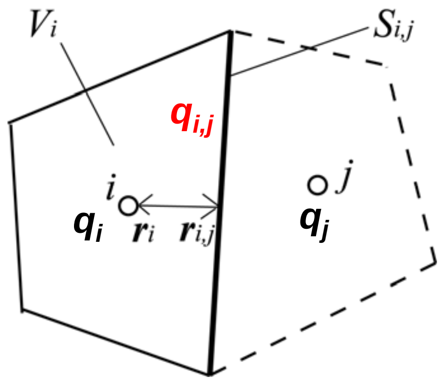
Inviscid Flux

Viscous Flux

$$q_{i,j} = q_i + \phi \nabla q_i \bullet (\mathbf{r}_{i,j} - \mathbf{r}_i)$$

Slope

Limiter [0,1]

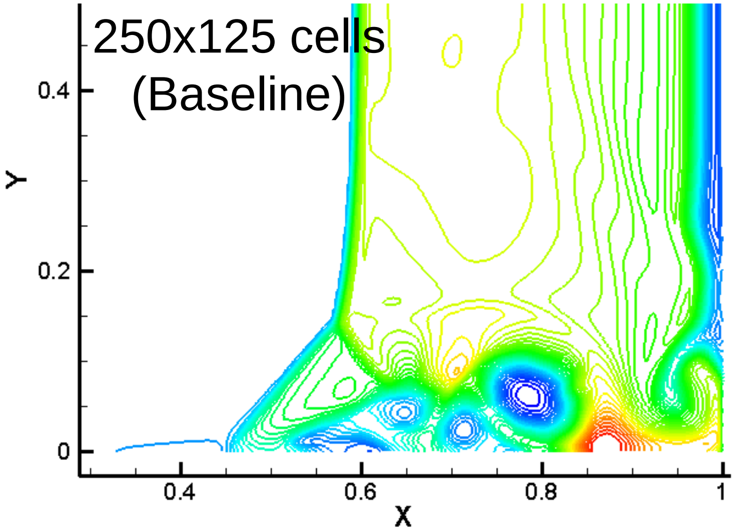


Dr. Hiroaki Nishikawa, one of *FUN3D* code developer, at National Institute of Aerospace:

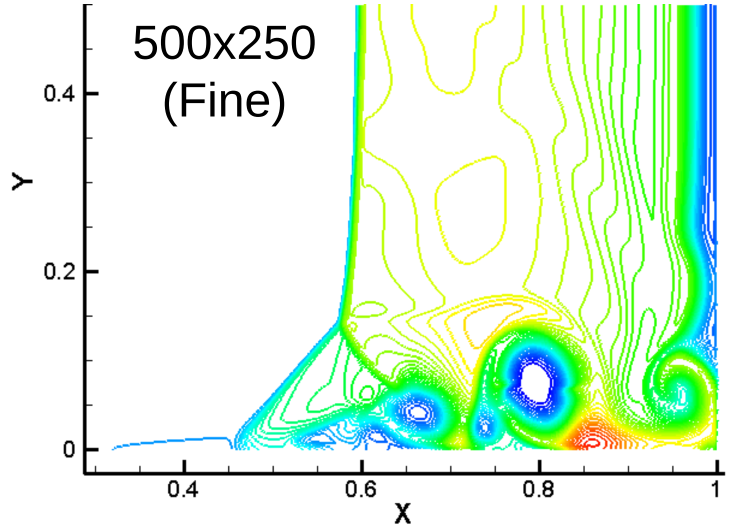
*“We **don’t use a limiter** for transonic simulations on a very smooth and beautiful mesh.”*

(Private Communication, 2013)

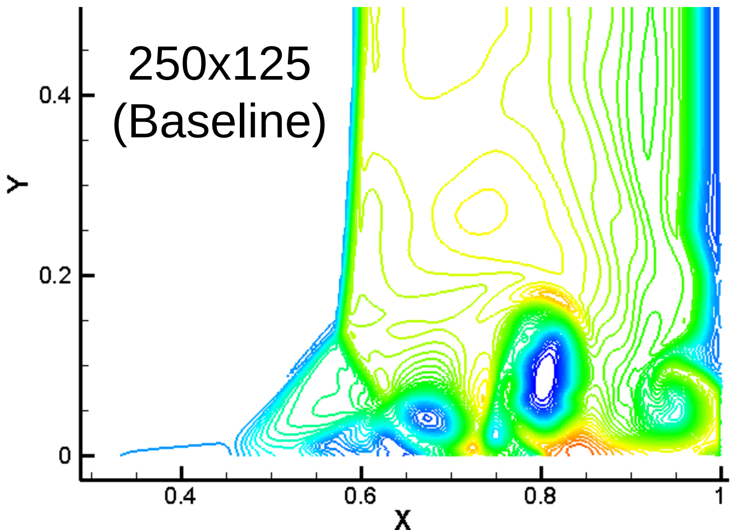
(Conventional, ***a priori***) Limited v.s. **Unlimited** Computations



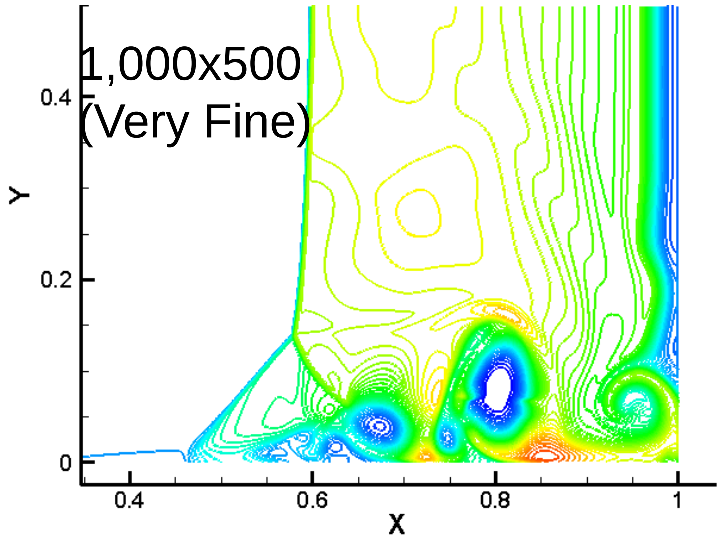
With Limiter



With Limiter



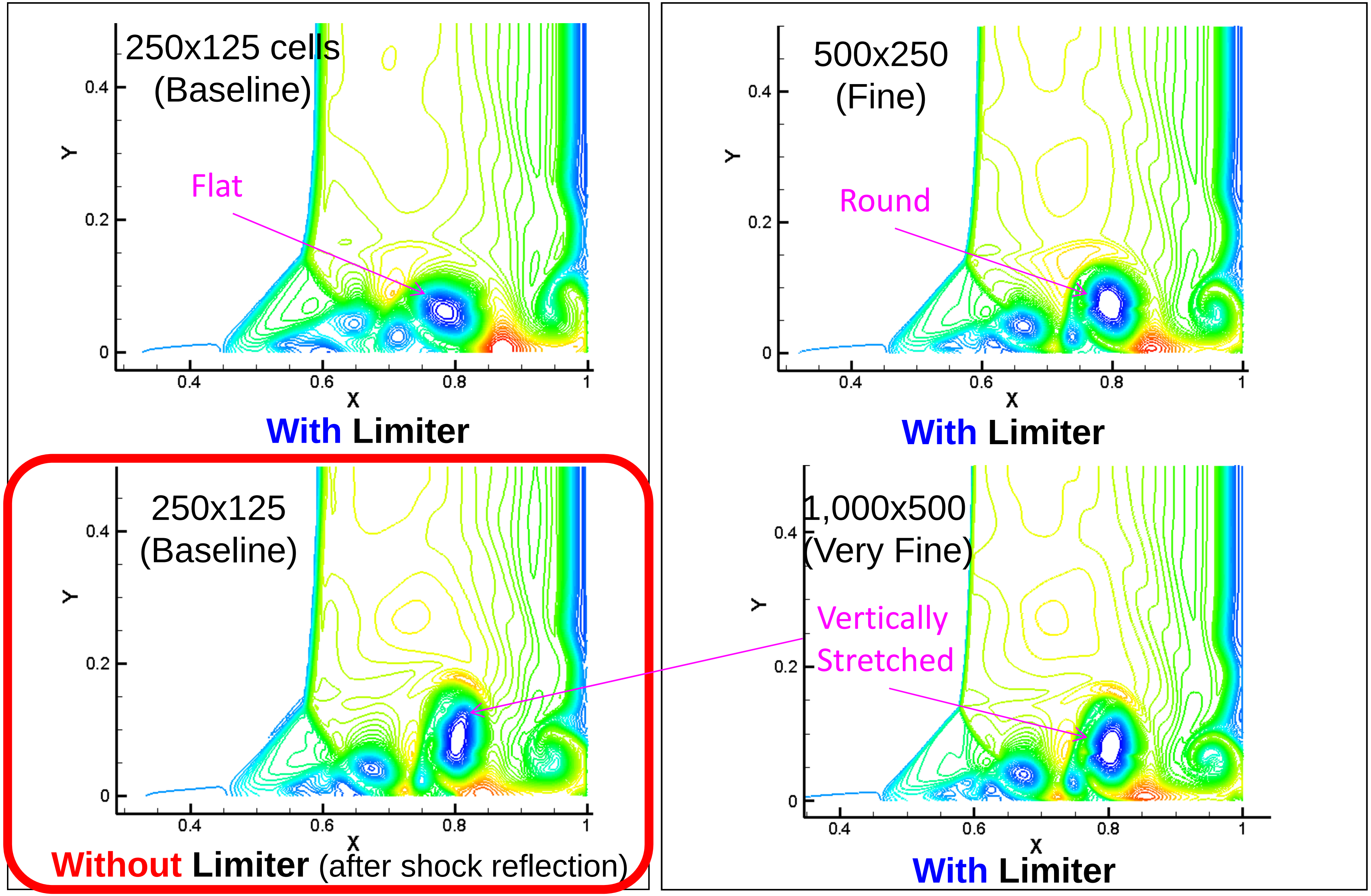
Without Limiter (after shock reflection)



With Limiter

AUSM$^+$-up, Viscous (Re=200), 2nd order in time and space

(Conventional, ***a priori***) Limited v.s. **Unlimited** Computations



(If stable,) **4x4=16 times equivalent resolution!**

a posteriori Limiting Procedure

Clain et al., *JCP*, 2011

- Compute Higher-Order (H.O.) Candidate Value $q_{i,j}$
- Then, Limiting: whether or not to adopt $q_{i,j}$

  - $q_{i,j}$ has no problem -> Adopted
  - $q_{i,j}$ is problematic -> Re-compute $q_{i,j}$ with Lower-order (L.O.), more stable method

$$q_{i,j} = q_i + \phi \nabla q_i \bullet \left(\mathbf{r}_{i,j} - \mathbf{r}_i\right)$$

Slope

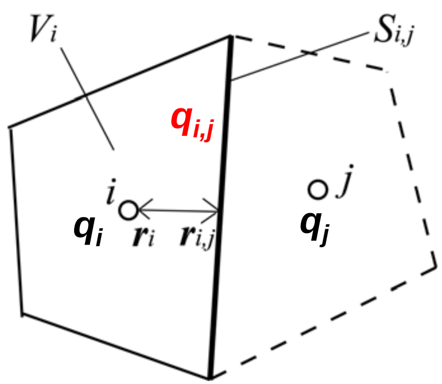
Limiter [0,1]

$$\underline{\underline{\mathbf{Q}_i^{n+1}}} = \mathbf{Q}_i^n - \frac{\Delta t_i}{V_i} \sum_j (\underline{\underline{\mathbf{F}_{i,j}}} - \underline{\underline{\mathbf{Fv}_{i,j}}}) S_{i,j}$$

Variable Vector at (n+1)-th Timestep

Inviscid Flux

Viscous Flux



MOOD (**M**ulti-dimensional **O**ptimal **O**rder **D**etection)

do

- Get Candidate Value $q_{i,j}$ with H.O. method

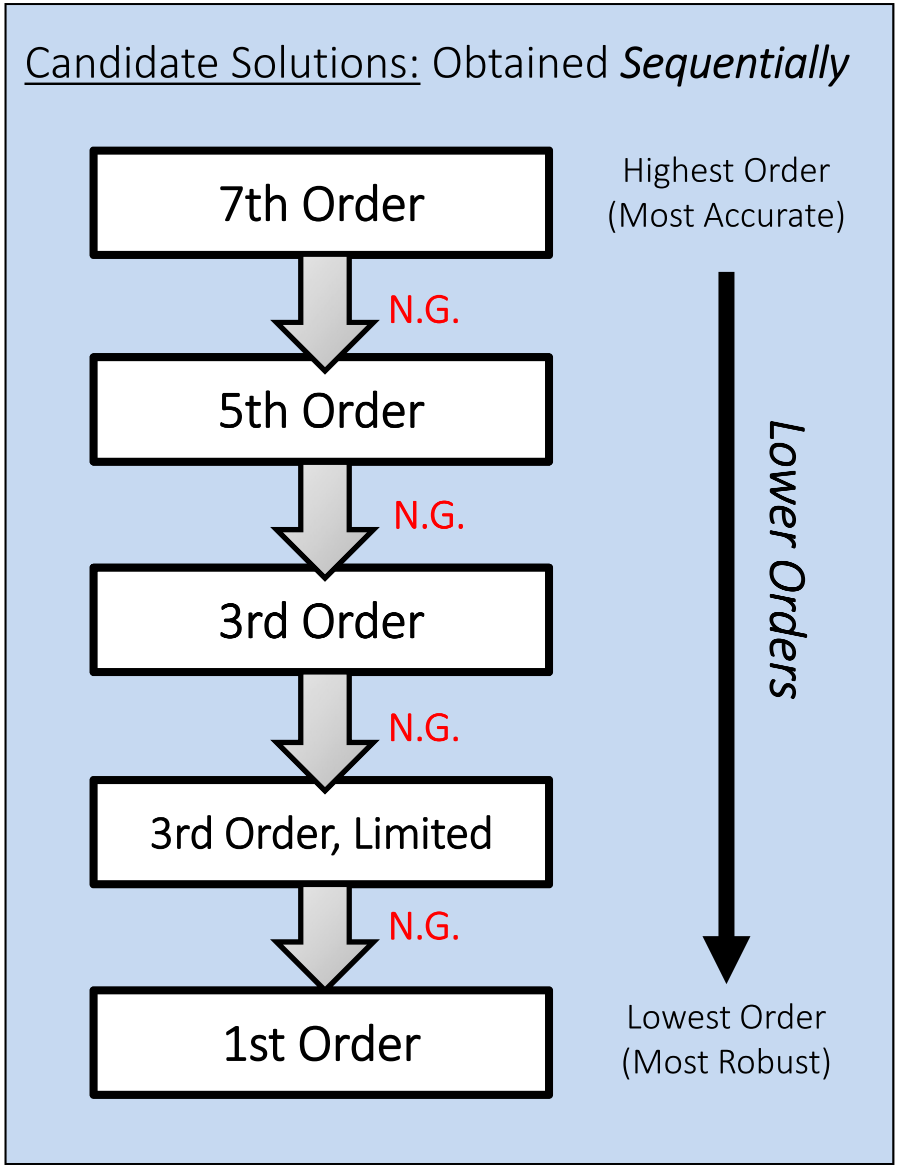
- Then, determine whether to adopt $q_{i,j}$

 - $q_{i,j}$ has no problem -> adopt

 - $q_{i,j}$ is problematic -> Go down to L.O. method and re-calculate $q_{i,j}$

end do

Repeated until satisfactory value is obtained
(-> extra cost and code complexity)



Clain et al., *JCP*, 2011

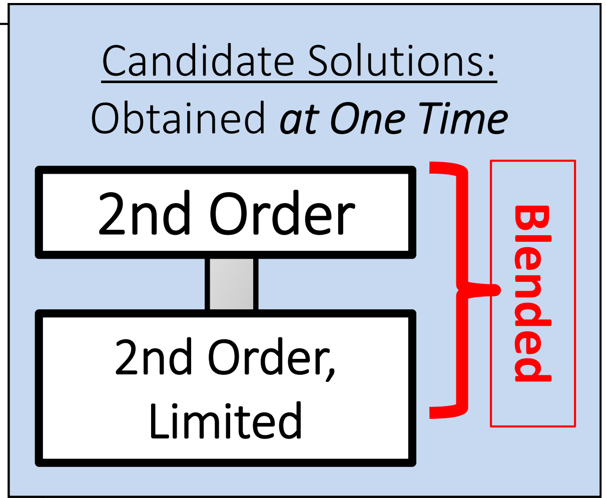
Proposed Method: Simple ***a posteriori*** Limiter

- Get Candidate Values $q_{i,j}^{unlim}$ and $q_{i,j}^{lim}$ <u>at one time</u>

- Then, determine which to use

 - $q_{i,j}^{unlim}$ has no problem -> Adopt $q_{i,j}^{unlim}$

 - $q_{i,j}^{unlim}$ is problematic -> Adopt $q_{i,j}^{lim}$



$$q_{i,j}^{unlim} = q_i + \nabla q_i \bullet (\mathbf{r}_{i,j} - \mathbf{r}_i)$$

$$q_{i,j}^{lim} = q_i + \phi \nabla q_i \bullet (\mathbf{r}_{i,j} - \mathbf{r}_i)$$

***a priori* Limiter [0,1]**

$$q_{i,j} = q_i + \phi \nabla q_i \bullet (\mathbf{r}_{i,j} - \mathbf{r}_i)$$

Slope

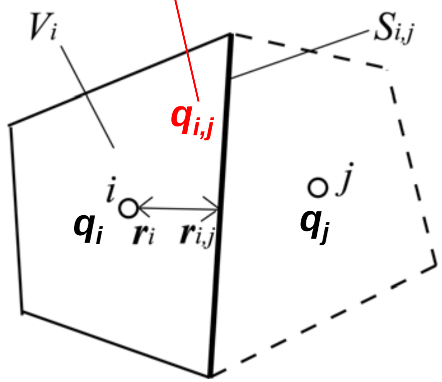
***a priori* Limiter [0,1]**

$$\underline{\underline{\mathbf{Q}_i^{n+1}}} = \mathbf{Q}_i^n - \frac{\Delta t_i}{V_i} \sum_j (\underline{\underline{\mathbf{F}_{i,j}}} - \underline{\underline{\mathbf{Fv}_{i,j}}}) S_{i,j}$$

Variable Vector at (n+1)-th Timestep

Inviscid Flux

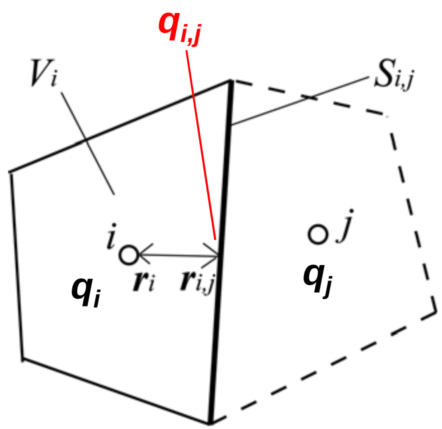
Viscous Flux



Criteria: How to choose from/blend q^{unlim} and q^{lim}

1. **Positivity:** Density and Pressure must be positive

$$\rho_{i,j}^{unlim} > 0,\ p_{i,j}^{unlim} > 0$$



2. **Discrete Maximum Principle (DMP):**
   Density at cell-interface must fall between values of cells at both sides

$$\min(\rho_i, \rho_j) < \rho_{i,j}^{unlim} < \max(\rho_i, \rho_j)$$

or Pressure ratio: 2 or lower

$$\boldsymbol{q}_{i,j} = \phi_G\, \boldsymbol{q}_{i,j}^{unlim} + (1-\phi_G)\, \boldsymbol{q}_{i,j}^{lim}$$

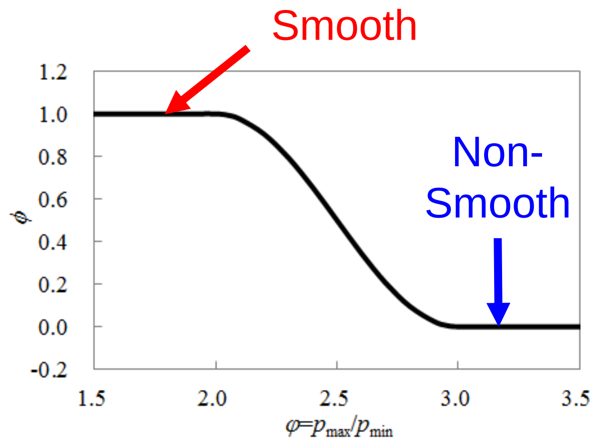
Gnoffo's
Auxiliary Limiter

Gnoffo, AIAA-2010-1271

$$\phi_G = \frac{1-\cos(\Phi\pi)}{2}, \quad \Phi = \min\left[1, \max\left(0, \frac{\varphi_{max}-\varphi}{\underbrace{\varphi_{max}}_{\fallingdotseq 3}-\underbrace{\varphi_{min}}_{\fallingdotseq 2}}\right)\right]$$

$$\varphi = \frac{p_{max}}{p_{min}},$$

$$p_{max} = \max\left(p^{nolim}, p_i, p_j\right), \quad p_{min} = \min\left(p^{nolim}, p_i, p_j\right)$$



Criteria: How to choose from/blend q^{unlim} and q^{lim}

1. Positivity: Density and Pressure must be positive

$$\rho_{i,j}^{unlim} > 0,\ p_{i,j}^{unlim} > 0$$

N

$: \phi_G = 0$

Y

2. Discrete Maximum Principle (DMP):
   Density at cell-interface must fall
   between values of cells at both sides

$$min(\rho_i, \rho_j) < \rho_{i,j}^{unlim} < max(\rho_i, \rho_j)$$

Y: $\phi_G = 1$

or Pressure ratio: 2 or lower

N

$$\boldsymbol{q}_{i,j} = \phi_G\, \boldsymbol{q}_{i,j}^{unlim} + (1-\phi_G)\, \boldsymbol{q}_{i,j}^{lim}$$

Gnoffo's
Auxiliary Limiter

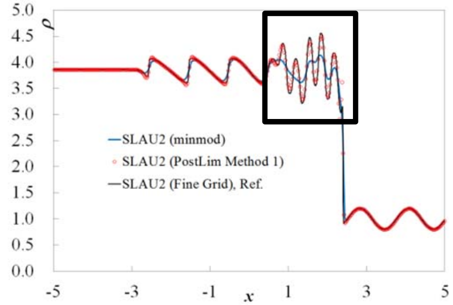
Gnoffo, AIAA-2010-1271

or

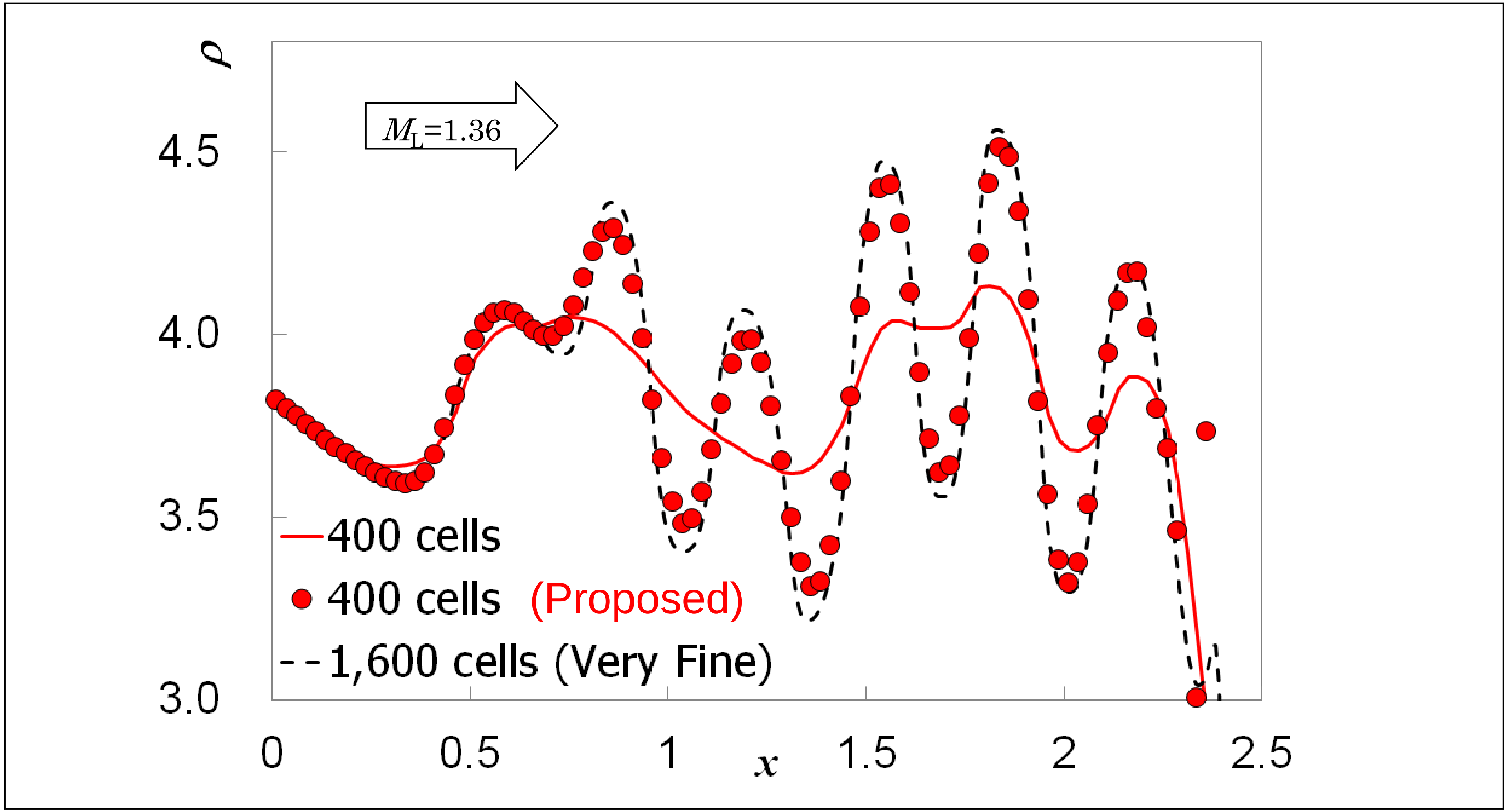
$$q_{i,j} = q_i + \phi_{final} \nabla q_i \bullet \left(\mathbf{r}_{i,j} - \mathbf{r}_i\right)$$

$$\phi_{final} = \phi_G + \left(1-\phi_G\right)\cdot \phi_{\lim}$$

Numerical Example #1: Shu-Osher's Problem (1D Shock/Entropy-Wave Interaction)



2nd-order in space and time, SLAU2, CFL≈0.68 (t=1.8)



➢ Equivalently 4 times resolution is achieved!

Numerical Example #2: Viscous Shock Tube

Re=200, 250x125 cells,
2^{nd}-order in space and time both,
200,000 steps (CFL≈ 0.52) (t=1)

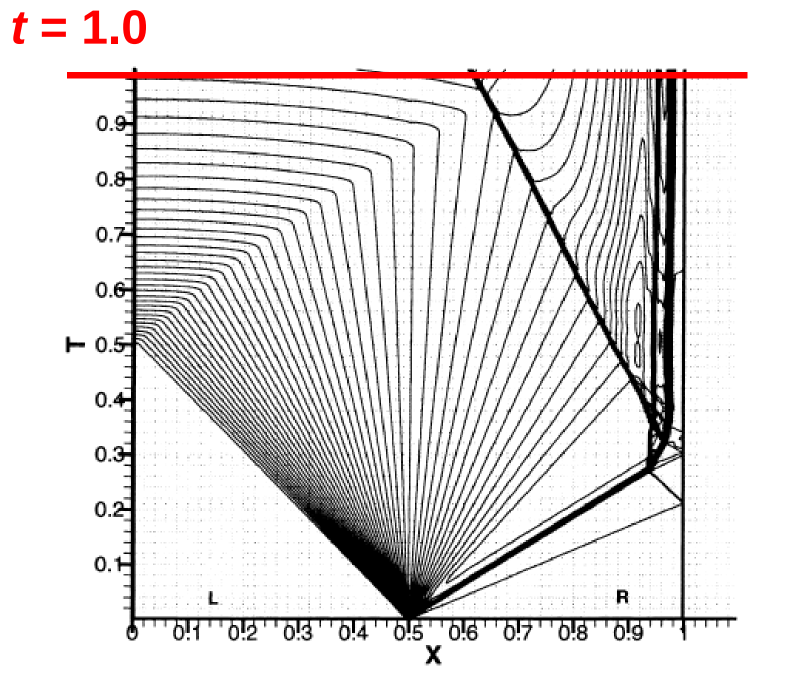
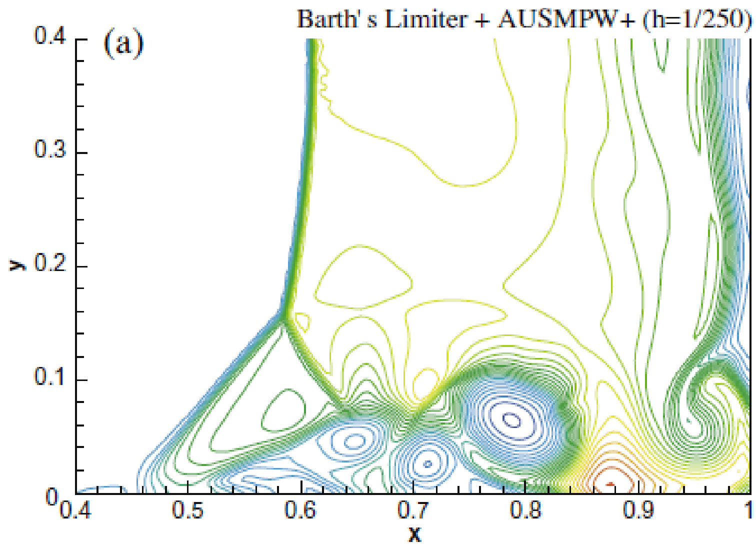
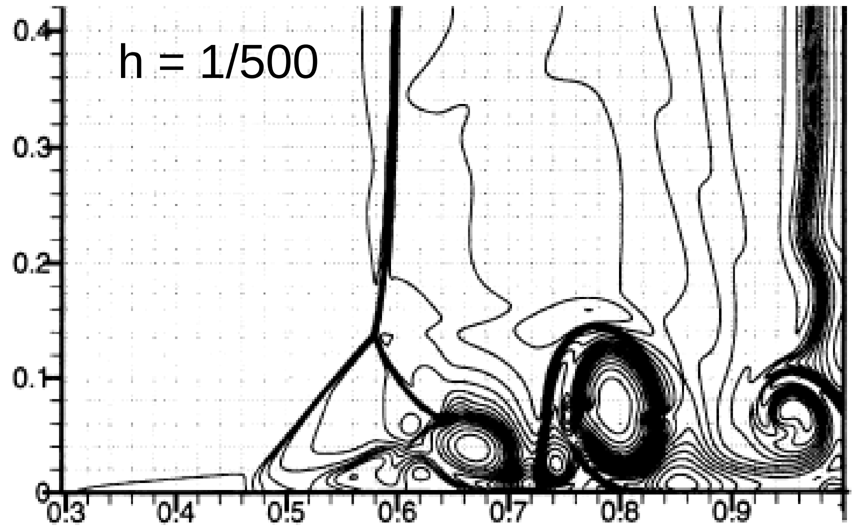


Fig. 9. x–t diagram of the density.

Inviscid Estimation (x-t diagram)
by Daru & Tenaud, CAF 2001



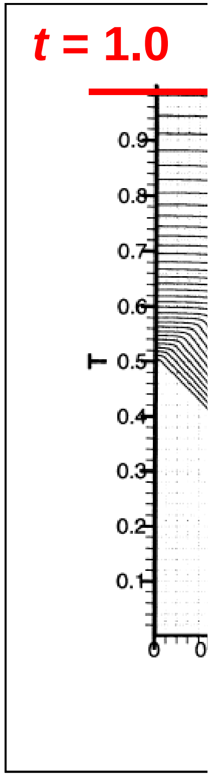
Kim et al., JCP 2010

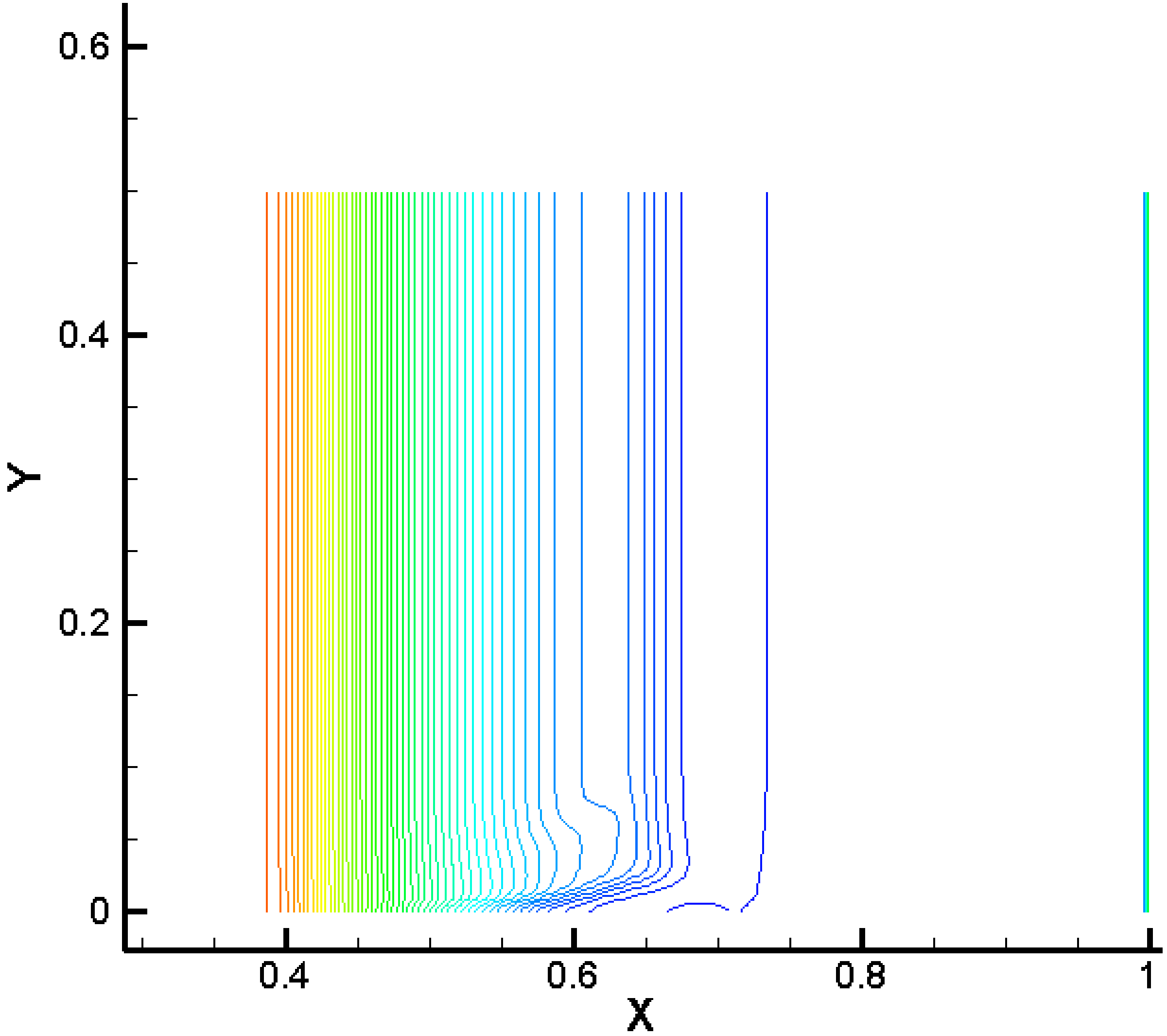


Daru & Tenaud, CAF 2001

Nume
Visc

Re
2nd-ord
200,00

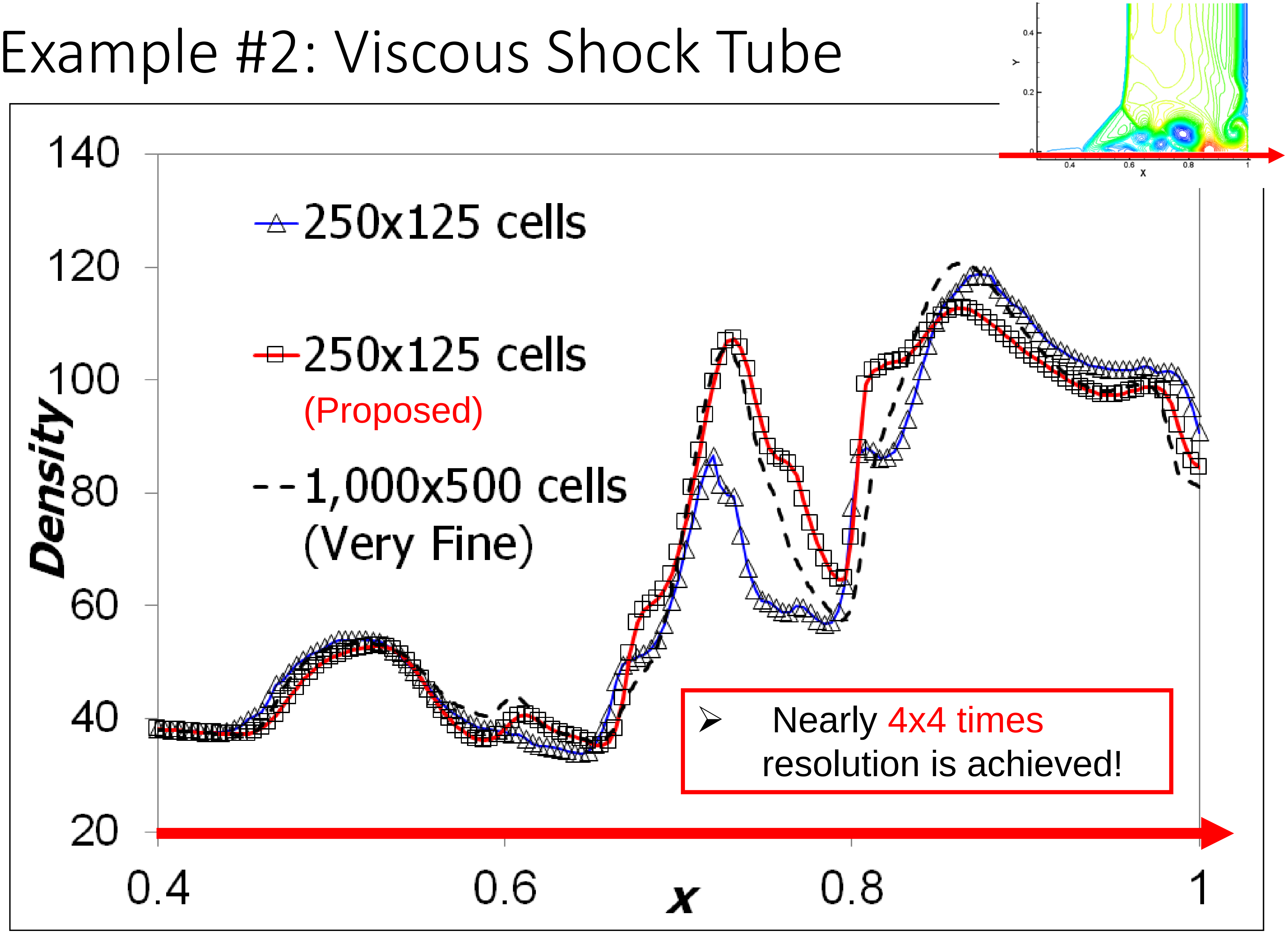




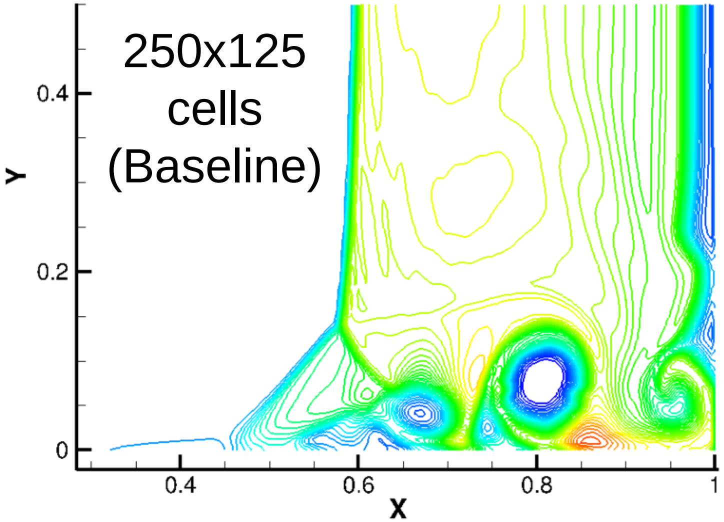
Invisci
by Daru & Tenaud, CAF 2001

Daru & Tenaud, CAF 2001

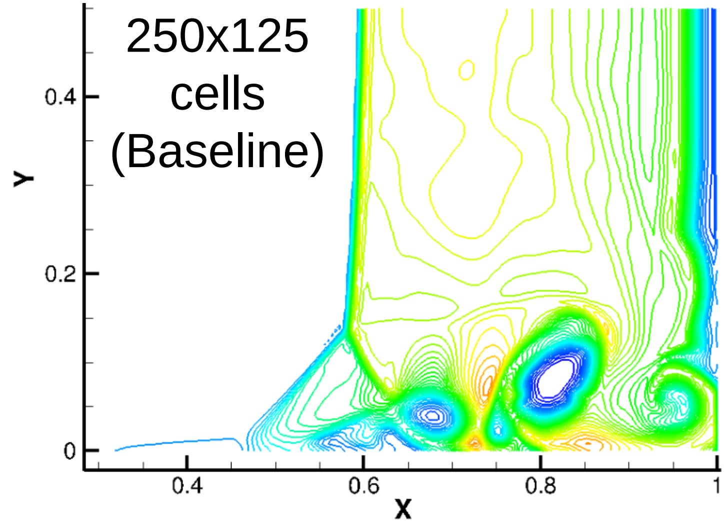
Example #2: Viscous Shock Tube



Example #2: Viscous Shock Tube

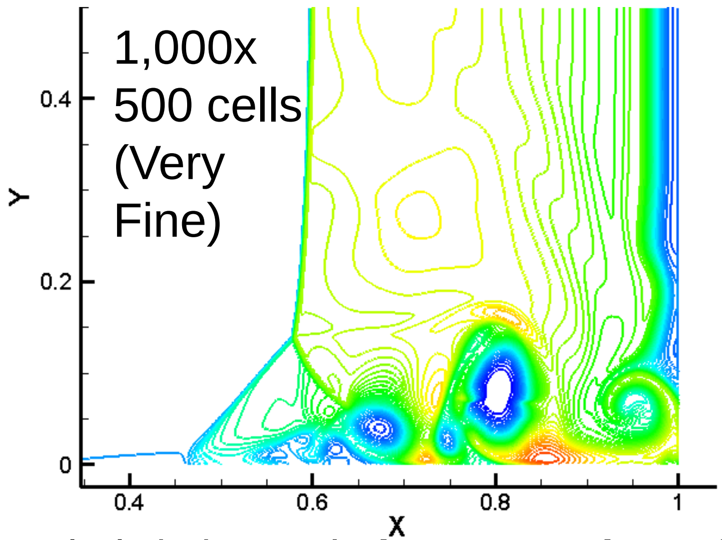


a priori Limiter Only



a posteriori Limiter

Closer to very fine solution

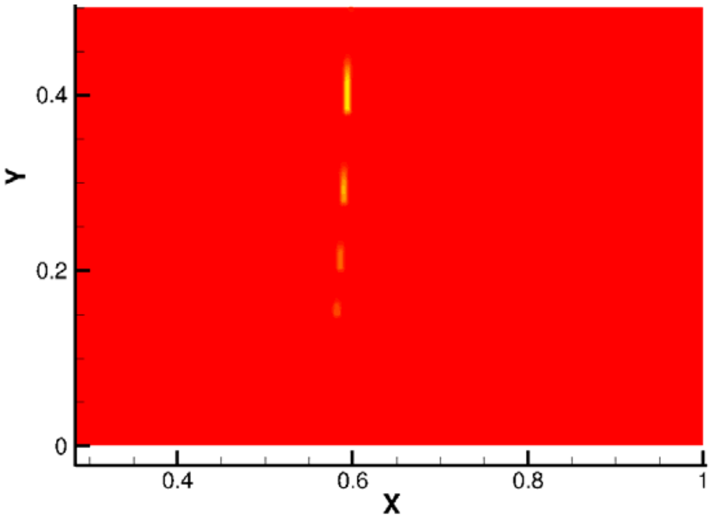


a priori Limiter Only **(MUSCL+minmod)**

AUSM$^+$-up, **Green-Gauss + MLP**

Example #2: Viscous Shock Tube

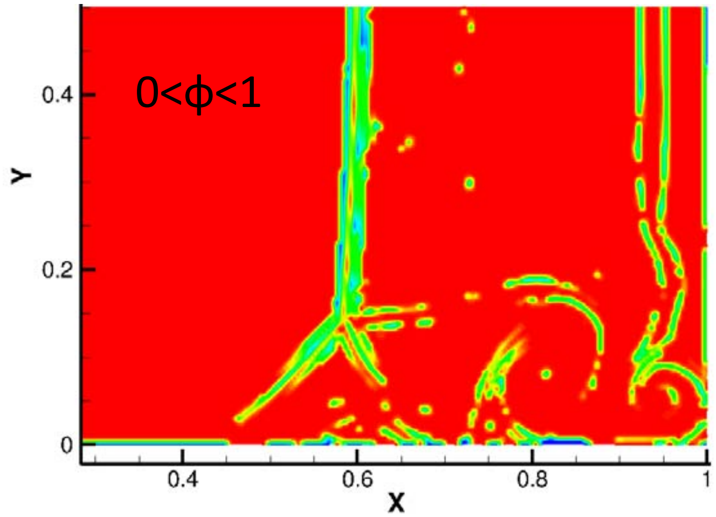
With *a posteriori* Limiter

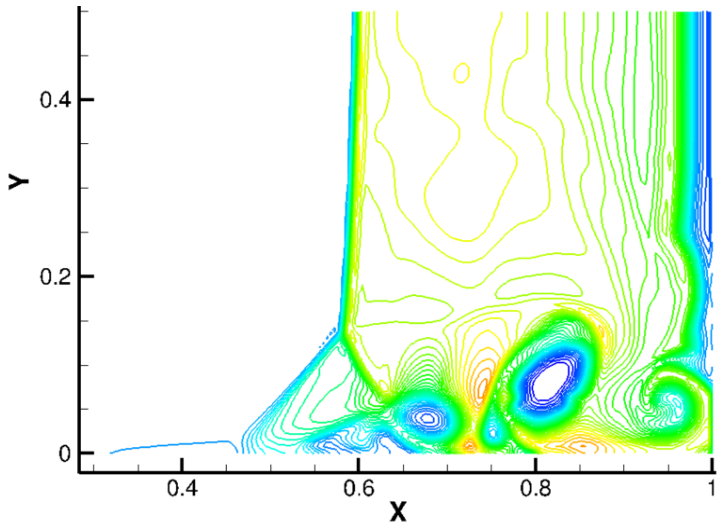


Limiter works at only small portions of shock!

AUSM$^+$-up, **Green-Gauss + MLP**

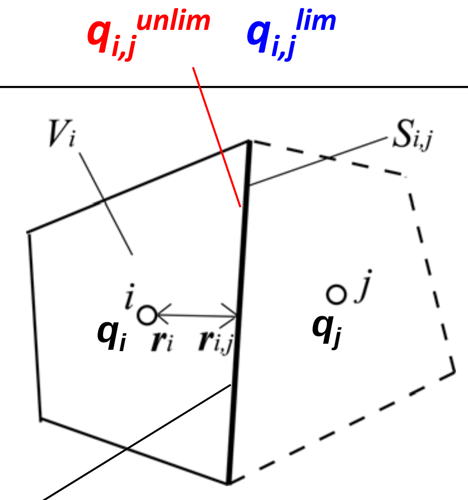
Activated region of *(a priori)* Limiter





Summary

- Simple *a posteriori* Limiter is proposed
  1. Get Candidate Values $q_{i,j}^{unlim}$ and $q_{i,j}^{lim}$ at One Time, without and with (*a priori*) Limiter, respectively
  2. Determine which to use, or blend them

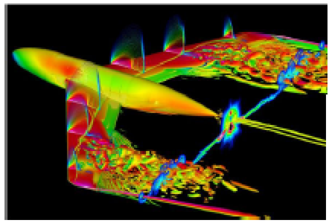


$$q_{i,j} = q_i + \phi_{final} \nabla q_i \bullet \left(\mathbf{r}_{i,j} - \mathbf{r}_i \right)$$

$$\phi_{final} = \phi_G + \left(1 - \phi_G\right) \cdot \phi_{\lim}$$

  - 1D: **4 times** equivalent resolution
  - 2D: nearly **4x4 times** resolution
  - Computational Overhead: Approx. 20% / timestep (which doesn't matter, compared with improved resolution) **-> Substantial Cost Reduction**

Brunet, AIAA 2008-4152



Future Work

- Multidimensional Consideration: Ducros Shock Sensor, etc
- Avoid If-sentence -> Further simplification
- 3D practical problems: Transonic Buffet Simulation etc.

Acknowledgments

- **Dr. Hiroaki Nishikawa** at National Institute of Aerospace let us know of unlimited computation even when shocks are present. This motivated us to initiate the present work.

- **Assoc. Prof. Ryoji Takagi and Assis. Prof. Taku Nonomura**, both at JAXA, provided us with valuable comments.