Turbulent boundary layer flow simulation using algebraic eddy viscosity model and wall functions

A.V.Severin, I.S.Menshov

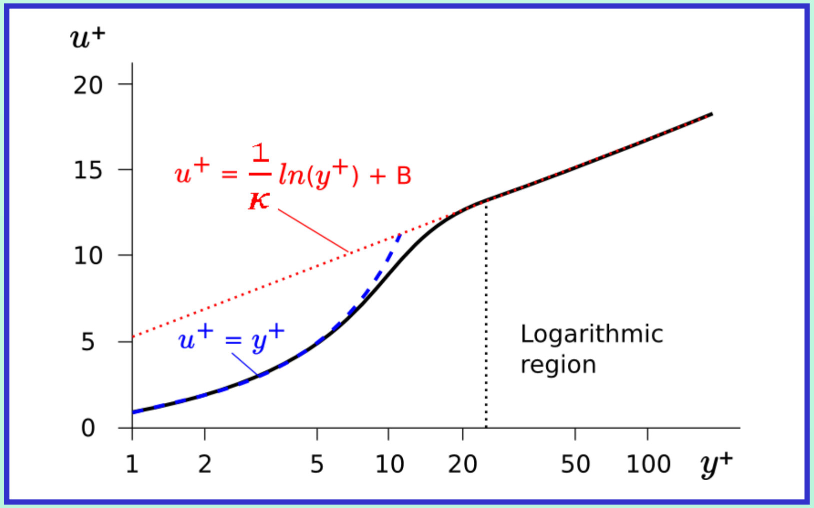
severin@kiam.ru, menshov@kiam.ru

Introduction

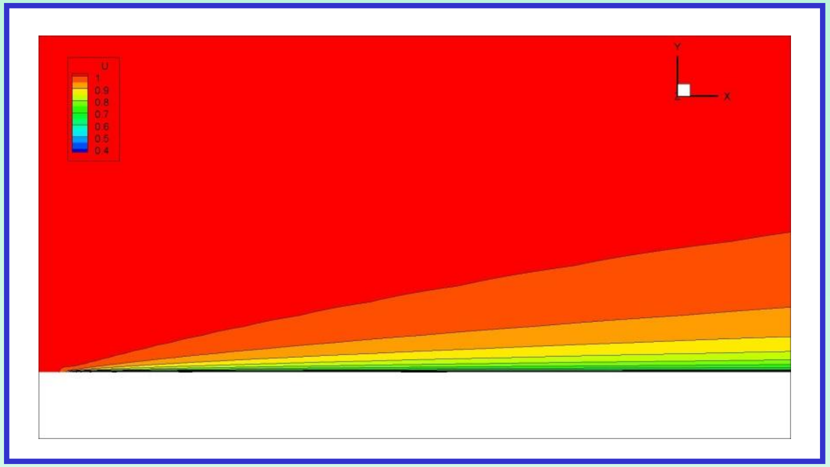
$$\mu_\tau = \rho l_m^2 \left| \overline{S} \right|$$

$$l_m = \min\left(C_{LES}\Delta, \kappa y \right)$$

+



=



Large Eddy Simulation

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{u}) = 0$$

$$\frac{\partial \rho \vec{u}}{\partial t} + div(\rho \vec{u} \circ \vec{u}) = -\nabla p + div(\tau + \tau_{tur})$$

$$\frac{\partial \rho E}{\partial t} + div(\rho \vec{u} H) = div[(\tau + \tau_{tur})\vec{u} + (\vec{q} + \vec{q}_{tur})]$$

ρ – density,

u – vector of velocity,

E – full energy to mass unit,

H – full enthalpy to mass unit,

τ – viscous stress tensor,

q – heat stream vector.

$$\vec{q}_{tur} = \lambda_{tur} \nabla T$$

$$(\tau_{tur})_{ij} = \mu_{tur}(\partial u_i / \partial x_j + \partial u_j / \partial x_i) - 2/3 \mu_{tur} \partial u_k / \partial x_k \delta_{ij}$$

Smagorinsky-Lilly subgrid viscosity model

$$\frac{\mu_\tau}{\rho} \sim \varepsilon^{1/3}\Delta^{4/3}$$

$$\varepsilon \sim \left|S\right|^3 \Delta^2$$

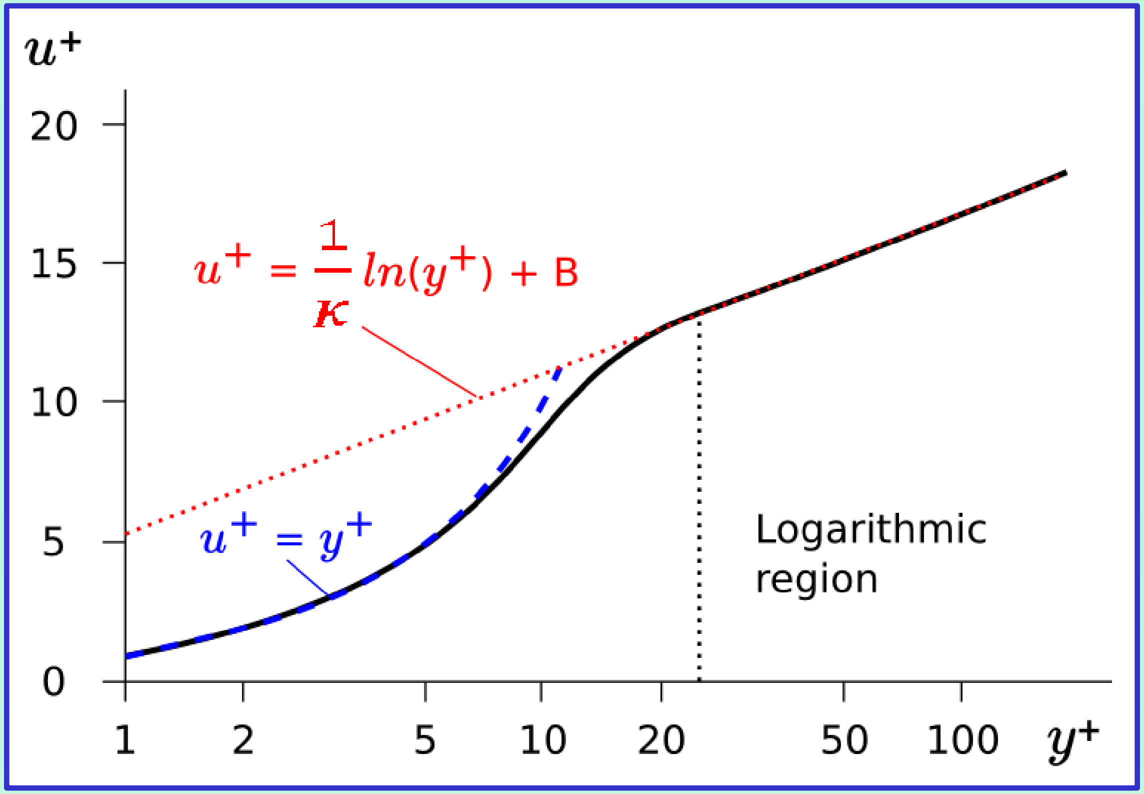
$$\mu_\tau = \rho\left(C_{LES}\Delta\right)^2\left|S\right|$$

$$\mu_\tau = \rho l_m^2 \left|\overline{S}\right|$$

$$l_m = \min\left(C_{LES}\Delta, \kappa y\right)$$

μ_τ – turbulent viscosity;
ρ – density;
ε – rate of dissipation;
l_m – mixing linear scale;
S – strain rate tensor;
$C_{LES} = 0.17$ – Smagorinsky constant;
Δ – typical cell size; (Here – the maximal diagonal of the cell.)
κ – Karman constant;
y – distance to the wall.

Log law of the wall



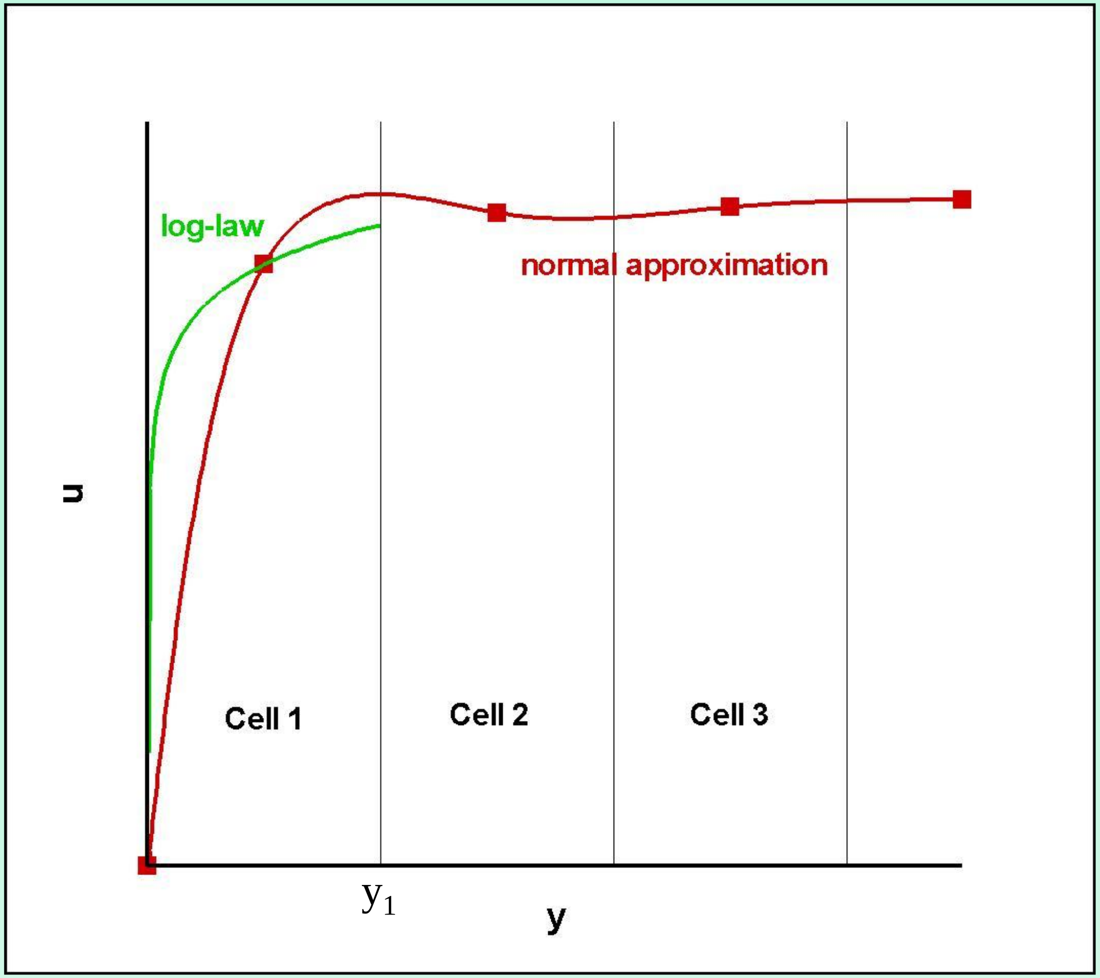
$$u = u_\tau u^+\left(y^+\right)$$

$$y^+ = y u_\tau / \nu;$$

u_τ – friction velocity, u^+ – universal function,
ν - molecular viscosity, y – distance to the wall.

Velocity profile approximation

$$U = \frac{1}{y_1}\int_0^{y_1} u(y)\,dy$$



Inverting of the log law

$$U = \frac{1}{y_1}\int_0^{y_1} u(y)dy$$

$$\int_0^{y_1} u(y)dy = u_\tau \int_0^{y_1} u^+(y^+)dy = u_\tau \frac{dy}{dy^+}\int_0^{y_1^+} u^+(y^+)dy^+ = \nu\int_0^{y_1^+} u^+(y^+)dy^+$$

$$Uy_1 = \nu\int_0^{y_1^+} u^+(y^+)dy^+$$

$$u^+ = \begin{cases} y^+ & in \quad y^+ < 2 \\ tabulated & in \quad 2 < y^+ < 60 \\ 2.5\ln\left(y^+ / 0.13\right) & in \quad y^+ > 60 \end{cases}$$

$$F(y_1^+) = \int_0^{y_1^+} u^+ dy^+ = \frac{Uy_1}{\nu} \longrightarrow y_1^+ = y_1^+(F)$$

Inverting of the log law

1.	$y^+ < 2$	$F < 2$
2.	$2 < y^+ < 60$	$2 < F < 696.68$
3.	$y^+ > 60$	$F > 696.68$

1. $$y_1^+ = \sqrt{2F}$$

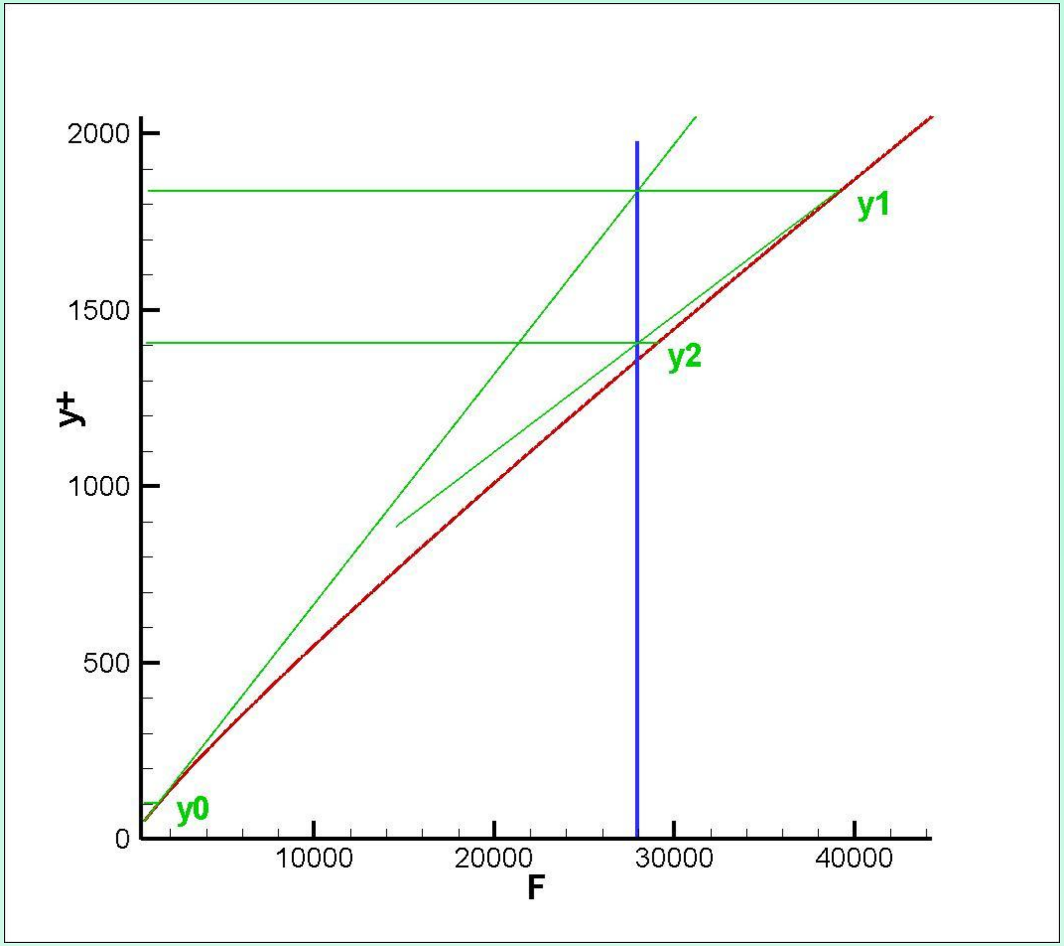
2.

F	2.	7.89	17.183	29.276	43.97	142.7	401.79	696.68
y^+	2.	4.	6.	8.	10.	20.	40	60.
dy^+/dF	0.5	0.256	0.185	0.149	0.125	0.0851	0.0706	0.0652

3.

$$u^+ = 2.5\ln\left(\frac{y^+}{0.13}\right) \qquad F = \int u^+ dy^+ = 2.5y^+\left(\ln\left(\frac{y^+}{0.13}\right) - 1\right) + C \qquad C = -73.50481$$

Newton iterations

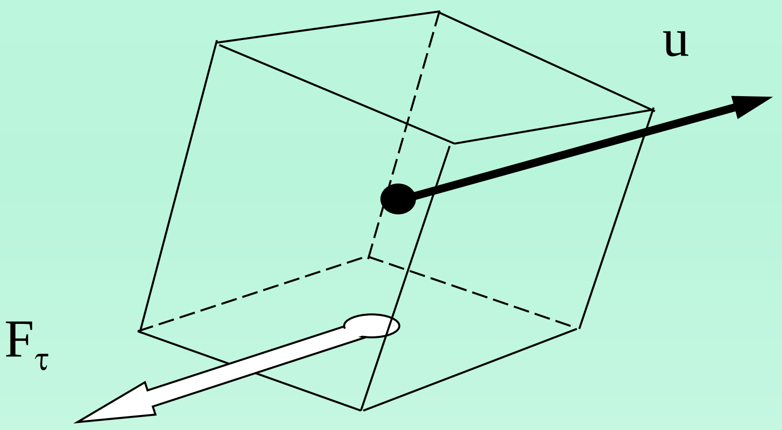


$$y^+_{i+1} = y^+_i + \frac{F - F_i}{u^+_i}$$

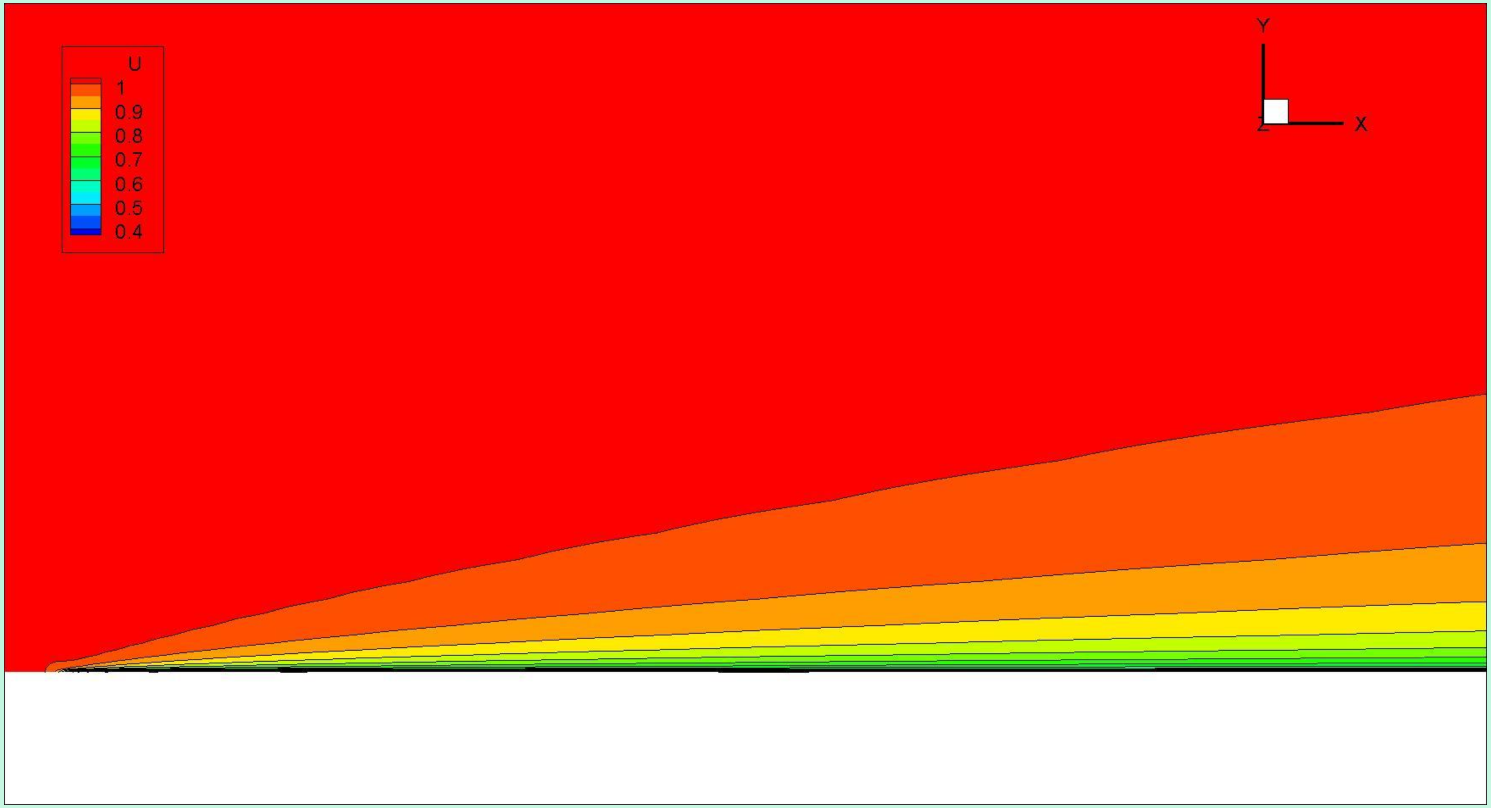
Boundary condition

$$u_\tau = \frac{y_1^+ \nu}{y_1}$$

$$\tau = u_\tau^2 \rho$$



Flat plate

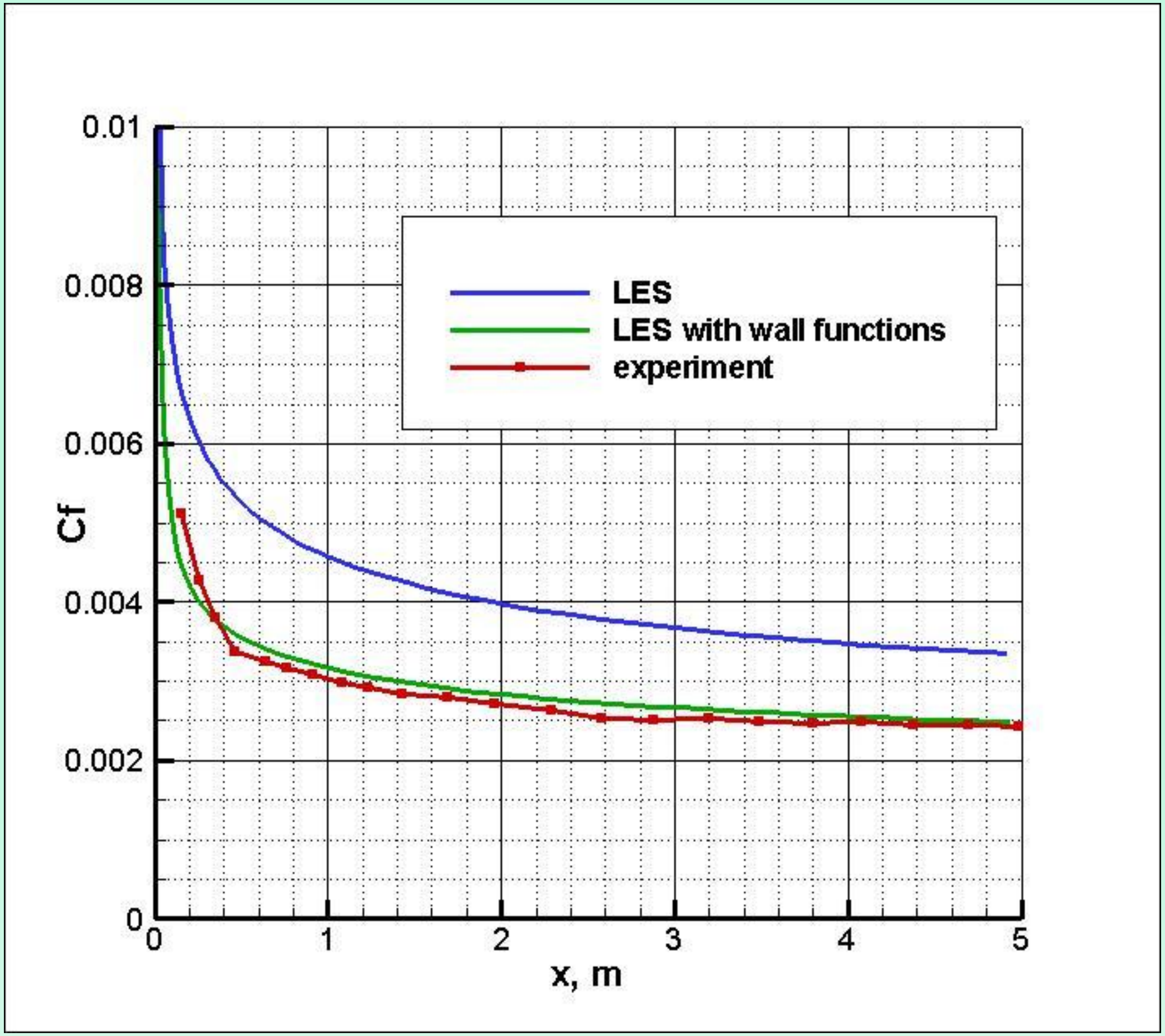


Boundary layer on the flat plate. M=0.1 Re=$2*10^6$ L = 5 m
Vertical coordinate is multiplied to 10.
Relative horizontal velocity is showed.

Flat plate

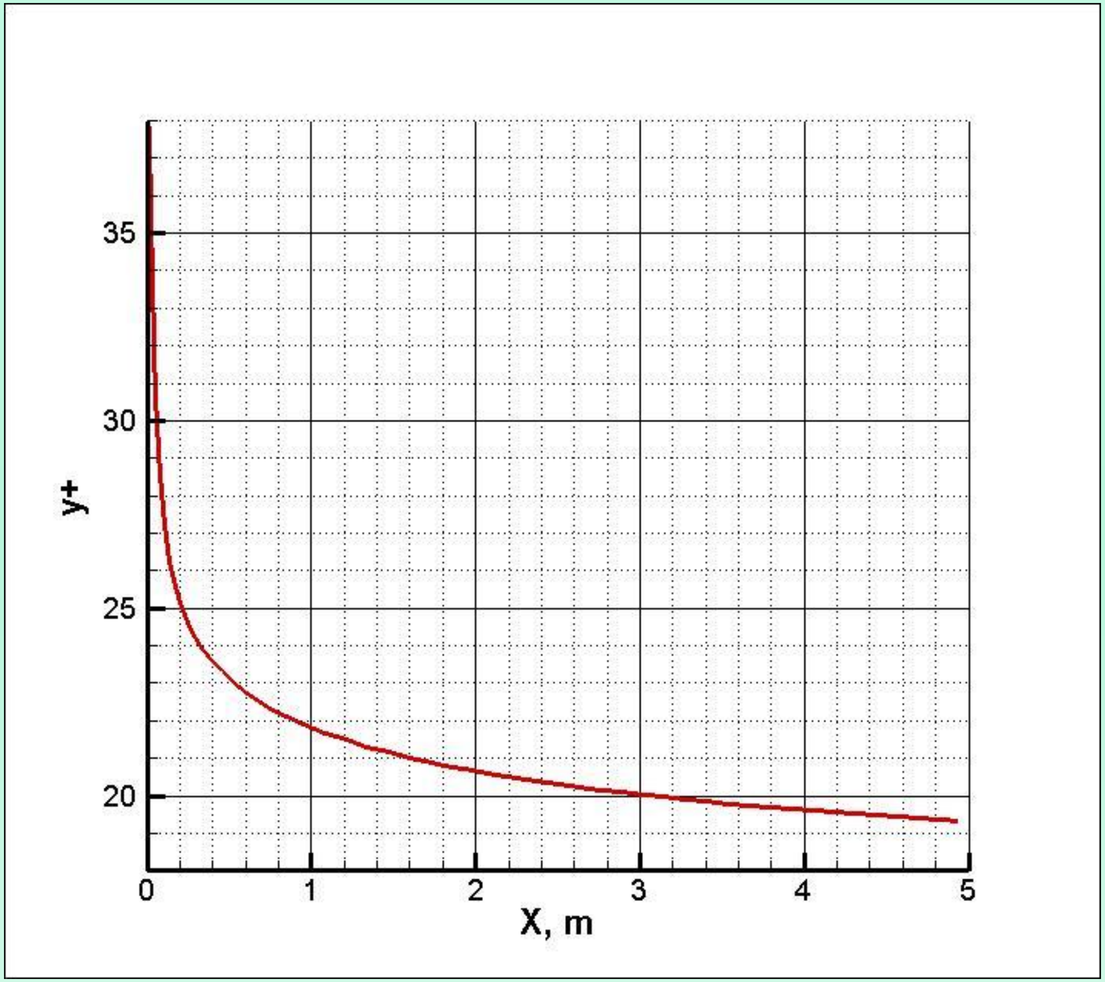
Boundary layer on the flat plate. M=0.1 Re=2*10^6 L = 5 m
Vertical coordinate is multiplied to 10.
Relative horizontal velocity near the edge of the plate.

Flat plate



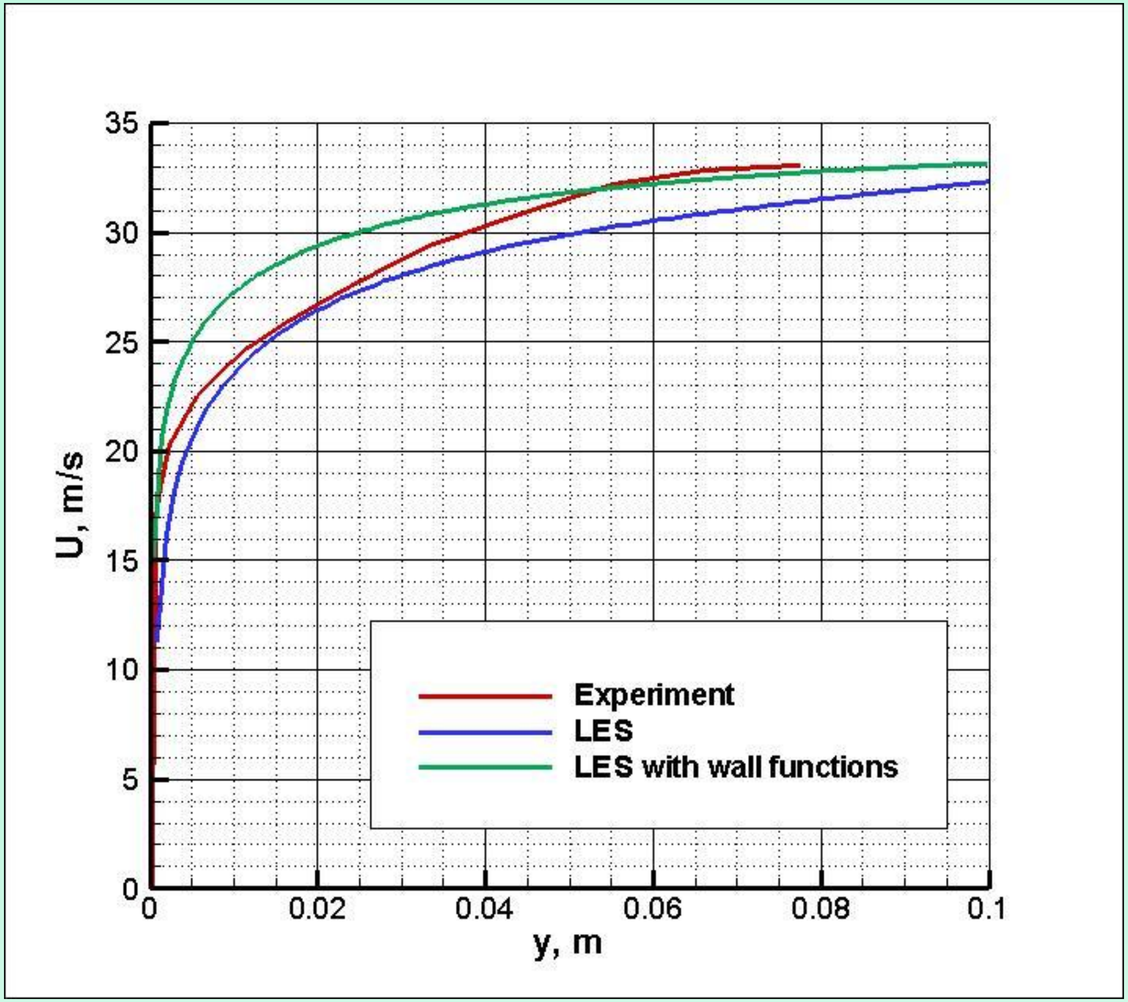
Friction coefficient on the flat plate.

Flat plate



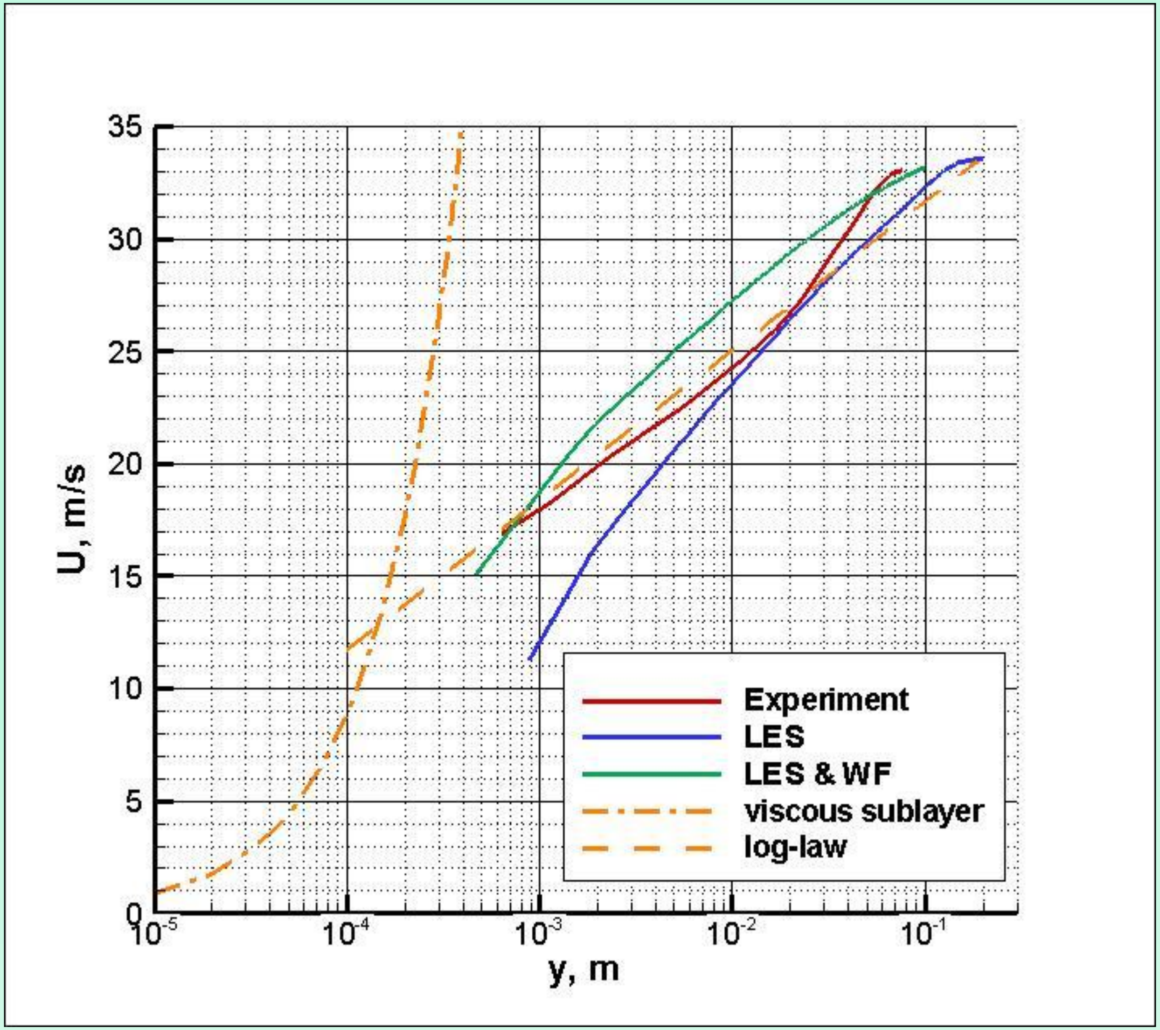
y^+ in the centers of near-wall cells.

Flat plate



Profile of velocities on x = 4.387

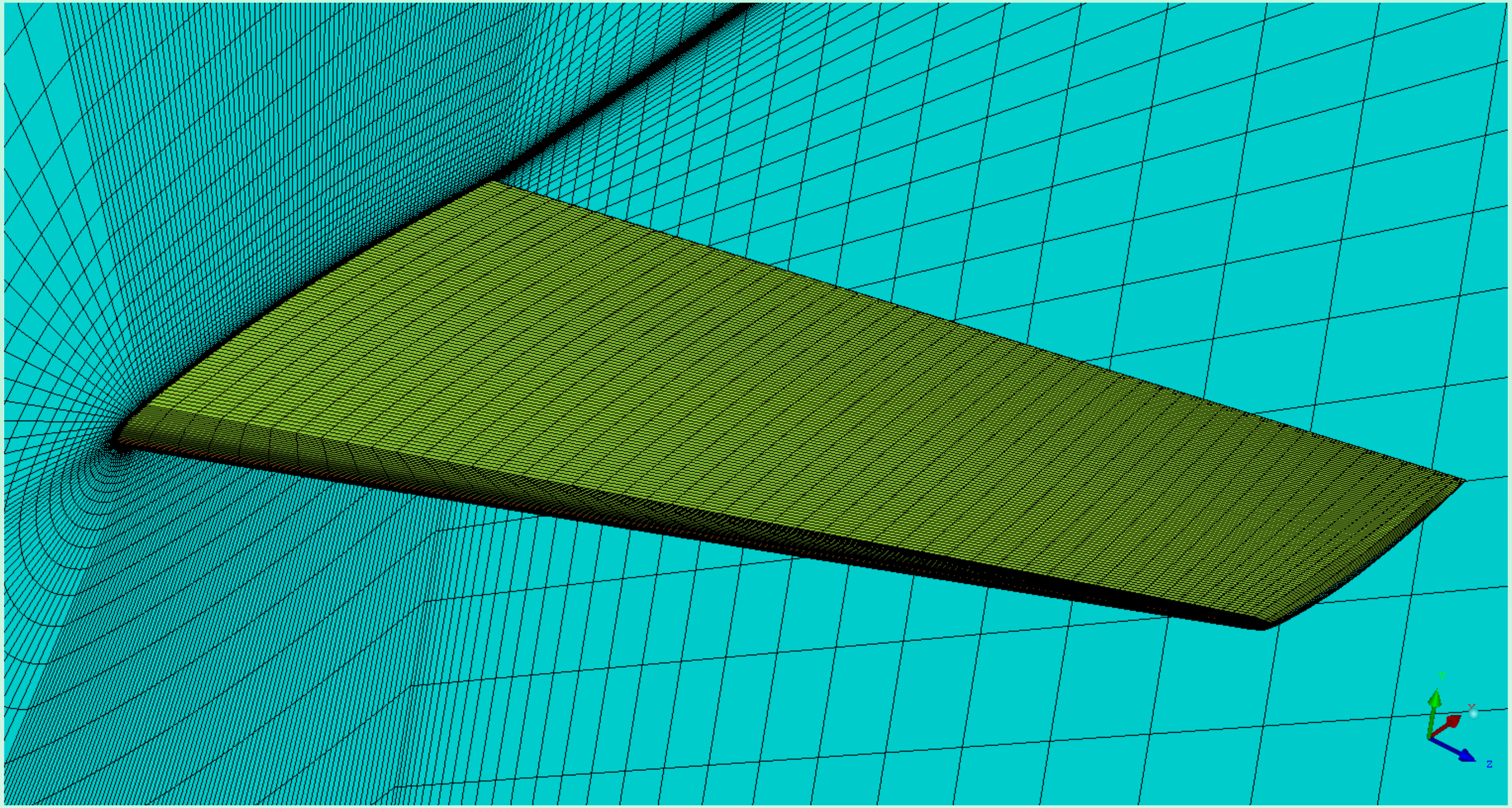
Flat plate



Profile of velocities on x = 4.387, log scale.

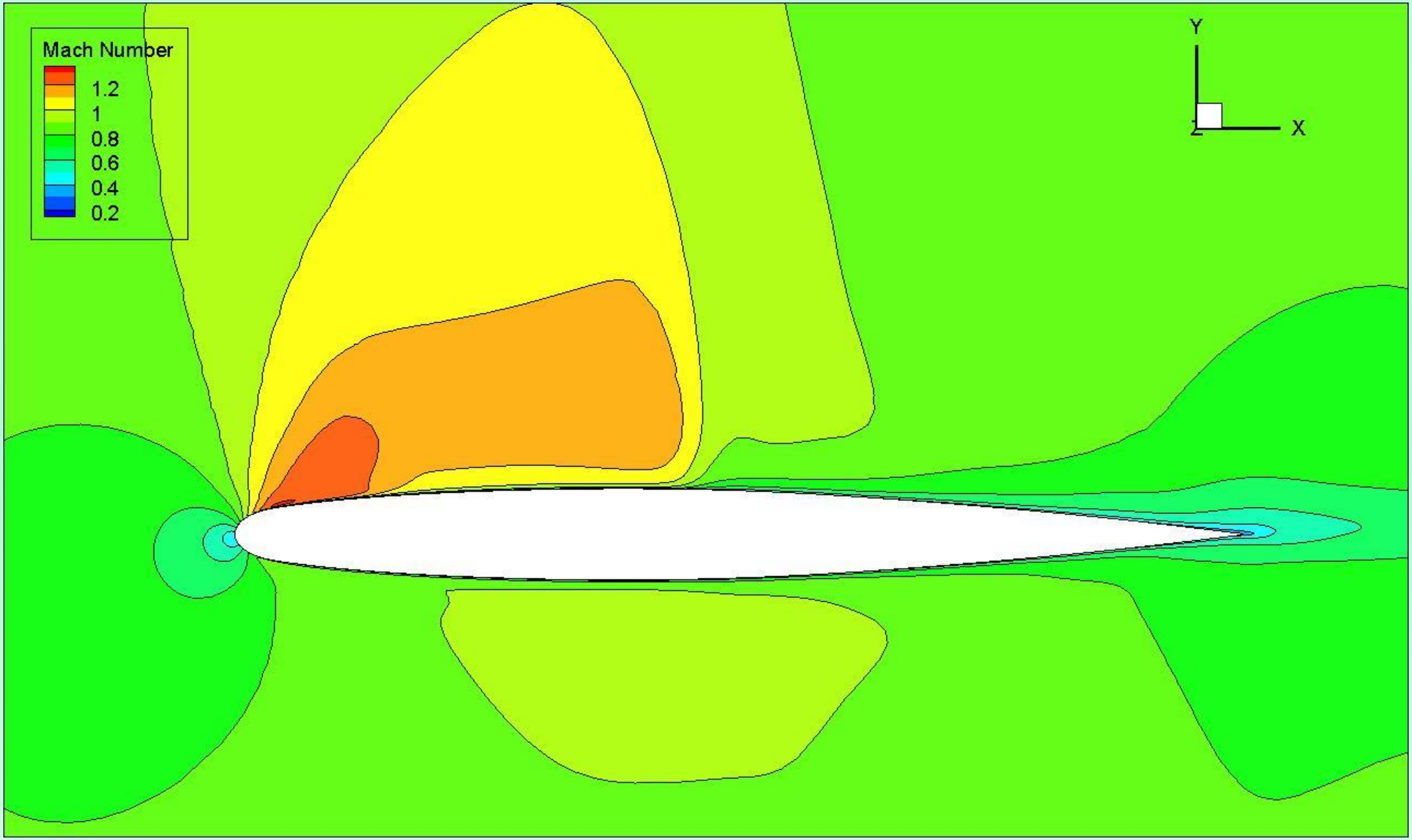
ONERA M6 wing

$M=0.8395$, $Re=11.72*10^6$, $\alpha=3.06^0$



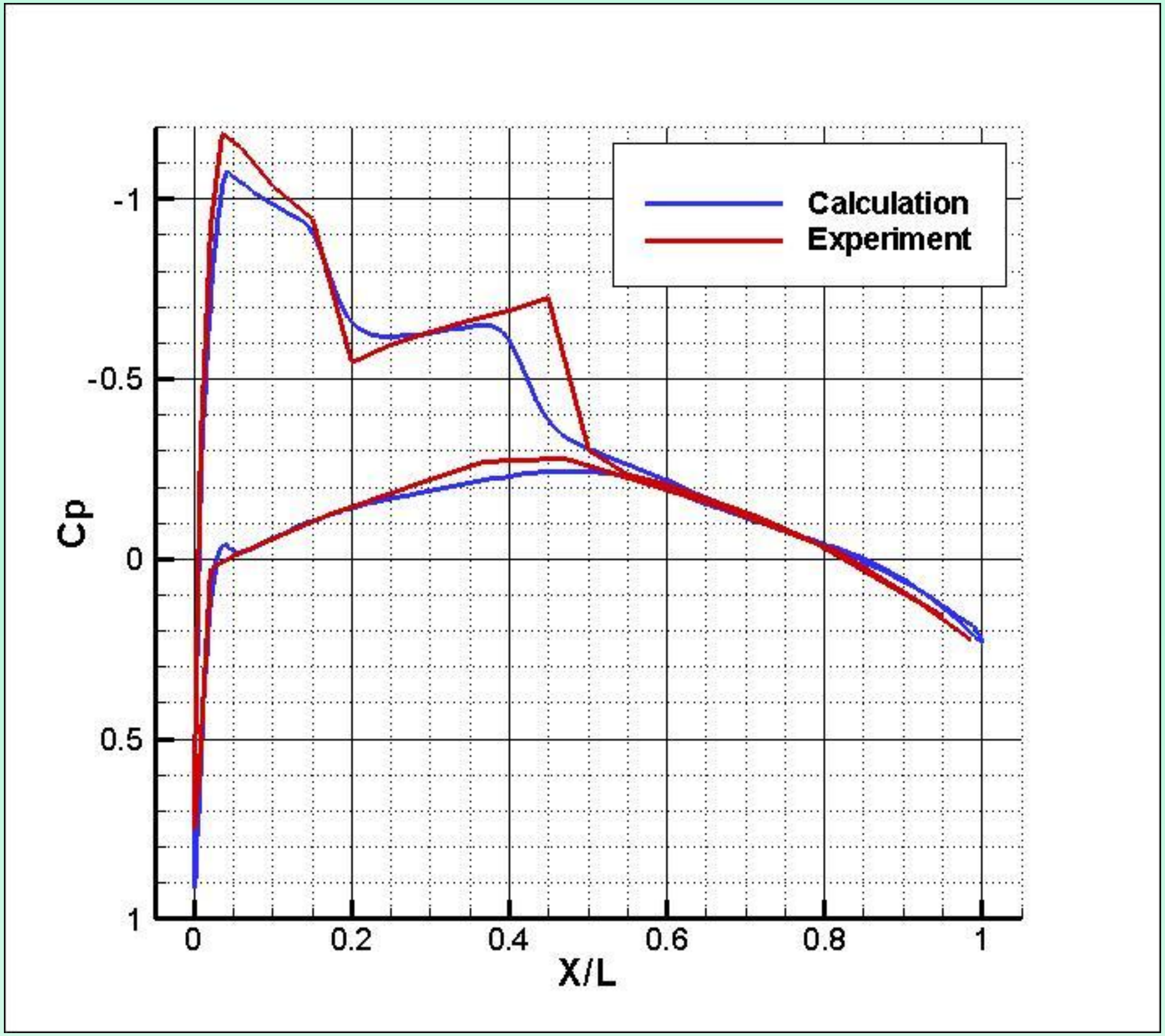
The grid.

ONERA M6 wing



Mach number on z=0.65b section.

ONERA M6 wing



Pressure coefficient on z=0.65b section.

References

1. I. Men’shov, Y. Nakamura, Hybrid Explicit–Implicit, Unconditionally Stable Scheme for Unsteady Compressible Flows // AIAA Journal, Vol. 42, No. 3, pp. 551-559, 2004.
2. Joseph Smagorinsky. General Circulation Experiments with the Primitive Equations. Monthly Weather Review, 1963.Vol. 91, pp. 99-164.
3. T. Knopp. On grid-independence of RANS predictions for aerodynamic flows using model-consistent universal wall-functions // European Conference on Computational Fluid Dynamics ECCOMAS CFD, 2006.