



SIMULATION OF REAL AERODYNASMICS PROBLEMS USING HIGHER-ACCURACY LOWER-COST SCHEME ON UNSTRUCTURED MESHES

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Introduction

- **EBR and WENO-EBR Schemes**
- In-House Code NOISEtte
- Verification
- Simulations





For solving real aerodynamics problems we need:

- 1) high accuracy (enough for engineering);
- 2) efficient **shock capturing techniques**;
- 3) efficient scalable **parallel algorithms**;
- 4) reasonable computational costs.

These items are interdependent and should be developed all in the same boat

We try to do this under the condition of **unstructured meshes** well suitable for the problems with complex geometry, typical for applications





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EBR (Edge-Based Reconstruction) Schemes =

vertex-centered schemes based on **quasi-1D edge-oriented reconstruction** of variables for **unstructured** meshes

1. Dervieux, A. and Debiez, C. Mixed element volume MUSCL methods with weak viscosity for steady and unsteady flow calculation. *Computer and Fluids* 29: 89-118, **1999**

2. Gourvitch, N., G. Rogé, I. Abalakin, A. Dervieux and T. Kozubskaya. A tetrahedral-based superconvergent scheme for aeroacoustics. *INRIA Report* 5212, **2004**

3. Abalakin, I., A. Dervieux and T. Kozubskaya. High Accuracy Finite Volume Method for Solving Nonlinear Aeroacoustics Problems on Unstructured Meshes. *Chinese Journal of Aeroanautics* 19: 97-104, **2006**

4. Koobus, B., F. Alauzet and A. Dervieux. Numerical algorithms for unstructured meshes, In: *Computational Fluid Dynamics*, edited by F. Magoulès, London, New-York: CRC Press, **2011**, p.131-203

5. Ilya Abalakin, Pavel Bakhvalov, Tatiana Kozubskaya. Edge-Based Methods in CAA. – In *"Accurate and Efficient Aeroacoustic Prediction Approaches for Airframe Noise"*, Lecture Series 2013-03, Ed. by C.Schram, R.Denos, E.Lecomte, von Karman Institute for Fluid Dynamics, **2013** (ISBN-13 978-2-87516-048-5)

6. Ilya Abalakin, Pavel Bakhvalov and Tatiana Kozubskaya. Edge-based reconstruction schemes for prediction of near field flow region in complex aeroacoustic problems, *International Journal of Aeroacoustics*, Vol.13, N 3&4, **2014**, p. 207-234





- Triangular and tetrahedral meshes, although extension to hybrid meshes has been done
- Method of lines for solving PDE
- Vertex-centered formulation

Node *i* is a center of cell of volume V_i

All the geometrical work (search for neighbors of 1st and 2nd levels, construction of cells, faces and normals, stencils, ...) is performed only once, at the preprocessing stage

• Conservation laws

result from the general formulation:

$$\left[\frac{\partial \mathbf{Q}}{\partial t}\right]_{i} + \frac{1}{V_{i}} \sum_{j \in N(i)} \mathcal{F}_{ij} S_{ij} = 0$$

N(i) — a set of faces of cell i containing node i(i.e. a set of edges going out of node i) \mathcal{F}_{ij} — flux through the cell face ij of square S_{ij} (projected to the normal of cell i)



• Edge-wise implementation:

The main loop – on edges with accumulating necessary integral sums in nodes

The scheme construction is reduced to the problem: **how to determine the fluxes** \mathcal{F}_{ii} =???



Riemann Solvers

Important remark:

Define the flux \mathcal{F}_{ij} through each face in a single point = midpoint of corresponding edge (i.e. the edge intersecting this face)

$$\mathcal{F}_{ij} = h\left(\mathbf{Q}_{ij}^{L}, \mathbf{Q}_{ij}^{R}\right) = \frac{\mathbf{F}\left(\mathbf{Q}_{ij}^{L}\right) + \mathbf{F}\left(\mathbf{Q}_{ij}^{R}\right)}{2} - \frac{\delta}{2} \left|\mathbf{A}_{ij}\right| \left(\mathbf{Q}_{ij}^{R} - \mathbf{Q}_{ij}^{L}\right) \quad \text{Roe}$$

$$\mathcal{F}_{ij} = h\left(\mathbf{F}_{ij}^{L}, \mathbf{F}_{ij}^{R}, \mathbf{Q}_{i}, \mathbf{Q}_{j}\right) = \frac{\mathbf{F}_{ij}^{L} + \mathbf{F}_{ij}^{R}}{2} - \frac{\delta}{2} \operatorname{sign} \mathbf{A}_{ij} \left(\mathbf{Q}_{i}, \mathbf{Q}_{j}\right) \left(\mathbf{F}_{ij}^{R} - \mathbf{F}_{ij}^{L}\right) \text{Huang}$$

$$\mathcal{F}_{ij} = h\left(\mathbf{F}_{ij}^{L}, \mathbf{F}_{ij}^{R}, \mathbf{Q}_{i}, \mathbf{Q}_{j}, \mathbf{Q}_{ij}^{L}, \mathbf{Q}_{ij}^{R}\right) = \frac{\mathbf{F}_{ij}^{L} + \mathbf{F}_{ij}^{R}}{2} - \frac{\delta}{2} \left|\mathbf{A}\left(\mathbf{Q}_{i}, \mathbf{Q}_{j}\right)\right|_{ij} \left(\mathbf{Q}_{ij}^{R} - \mathbf{Q}_{ij}^{L}\right) \text{Hybrid}$$

$$\mathcal{F}_{ij} = \left(\mathbf{F}^{+}\right)_{ij}^{L} + \left(\mathbf{F}^{-}\right)_{ij}^{R}, \mathbf{F} = \mathbf{F}^{+} + \mathbf{F}^{-} \quad \text{Flux splitting methods}$$

$$(\bullet)^{L/R} - \text{reconstruction of mesh function from the left (L) or from the right (R) of cell face$$

$$\mathbf{F} = \mathbf{F}_x n_x + \mathbf{F}_y n_y + \mathbf{F}_z n_z, \quad \mathbf{A} = \frac{\partial \mathbf{F}}{\partial \mathbf{Q}} = \mathbf{S} \mathbf{A} \mathbf{S}^{-1}$$

Note, if $(\bullet)^{L} = (\bullet)^{R} = ((\bullet)_{i} + (\bullet)_{j})/2$

we obtain the 2nd-order vertex-centered (Gauss-Green) scheme for barycentric cells

If we like to gain **more accuracy** we should define the reconstructed variables $(\bullet)^{L} = ???$ and $(\bullet)^{R} = ???$ Face



EBR and WENO-EBR Schemes





EBR5 scheme (m=4)

EBR3 scheme (m=2)

$$\beta_{ij,1}^{L} = -\frac{1}{15} \frac{\Delta r_{7/2}}{\Delta r_{3/2}}, \quad \beta_{ij,2}^{L} = \frac{11}{30} \frac{\Delta r_{7/2}}{\Delta r_{5/2}}, \quad \beta_{ij,3}^{L} = \frac{4}{5} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,4}^{L} = -\frac{1}{10} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,1}^{L} = \frac{1}{3} \frac{\Delta r_{7/2}}{\Delta r_{5/2}}, \quad \beta_{ij,2}^{L} = \frac{2}{3} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,2}^{L} = \frac{2}{3} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,2}^{L} = \frac{1}{30} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,3}^{R} = -\frac{1}{10} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,4}^{R} = -\frac{1}{15} \frac{\Delta r_{7/2}}{\Delta r_{11/2}}, \quad \beta_{ij,1}^{R} = \frac{2}{3} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,2}^{R} = \frac{1}{30} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,4}^{R} = -\frac{1}{15} \frac{\Delta r_{7/2}}{\Delta r_{11/2}}, \quad \beta_{ij,1}^{R} = \frac{2}{3} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,2}^{R} = \frac{1}{3} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,4}^{R} = -\frac{1}{15} \frac{\Delta r_{7/2}}{\Delta r_{11/2}}, \quad \beta_{ij,1}^{R} = \frac{2}{3} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,2}^{R} = \frac{1}{3} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,2}^{R} = \frac{1}{3} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,2}^{R} = \frac{1}{3} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,4}^{R} = -\frac{1}{15} \frac{\Delta r_{7/2}}{\Delta r_{11/2}}, \quad \beta_{ij,1}^{R} = \frac{2}{3} \frac{\Delta r_{7/2}}{\Delta r_{7/2}}, \quad \beta_{ij,2}^{R} = \frac{1}{3} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,3}^{R} = \frac{1}{3} \frac{\Delta r_{7/2}}{\Delta r_{9/2}}, \quad \beta_{ij,3}^{R} = \frac{1}{3$$

WENO-EBR scheme (for *L*-reconstruction – from the left)

- On the 5-points stencil we can build **three** *L*-reconstructions of the 2nd-order (m=2). Apply them to characteristics variables $|\mathbf{S}^{-1}\mathbf{Q}|_{ii}$ Local numeration $\mathcal{L}_{ij}^{L^{(1)}} = \mathbf{S}_{ij}^{-1} \mathbf{Q}_i + \mathbf{S}_{ij}^{-1} \frac{\Delta r_{7/2}}{2} \left(-\frac{1}{3} \frac{\Delta \mathbf{Q}_{3/2}}{\Delta r_{3/2}} + \frac{5}{3} \frac{\Delta \mathbf{Q}_{5/2}}{\Delta r_{5/2}} \right) = \mathbf{S}_{ij}^{-1} \mathbf{Q}_i + SlopeL(1)$ **Global numeration** $\mathcal{L}_{ij}^{L^{(2)}} = \mathbf{S}_{ij}^{-1} \mathbf{Q}_i + \mathbf{S}_{ij}^{-1} \frac{\Delta r_{7/2}}{2} \left(\frac{1}{3} \frac{\Delta \mathbf{Q}_{5/2}}{\Delta r_{5/2}} + \frac{2}{3} \frac{\Delta \mathbf{Q}_{7/2}}{\Delta r_{7/2}} \right) = \mathbf{S}_{ij}^{-1} \mathbf{Q}_i + SlopeL(2)$ $\mathcal{L}_{ij}^{L^{(3)}} = \mathbf{S}_{ij}^{-1}\mathbf{Q}_{i} + \mathbf{S}_{ij}^{-1}\frac{\Delta r_{7/2}}{2} \left(\frac{4}{3}\frac{\Delta \mathbf{Q}_{7/2}}{\Delta r_{7/2}} - \frac{1}{3}\frac{\Delta \mathbf{Q}_{9/2}}{\Delta r_{0/2}} \right) = \mathbf{S}_{ij}^{-1}\mathbf{Q}_{i} + SlopeL(3) \quad \mathbf{S}_{ij}^{-1} = \mathbf{S}_{ij}^{-1}\left(\mathbf{Q}_{i}, \mathbf{Q}_{j}\right)$
- Weighted combination of three reconstructions in accordance to classical FD WENO method

$$\mathcal{L}_{ij}^{L} = \sum_{k=1}^{3} \omega_{k}^{L} \mathcal{L}_{ij}^{L^{(k)}}, \quad \omega_{k}^{L} = \frac{\sigma_{k}^{L}}{\sigma_{1}^{L} + \sigma_{2}^{L} + \sigma_{3}^{L}} \quad \sigma_{k}^{L} = \frac{\Omega^{(k)}}{\left(10^{-10} + IS_{k}^{L}\right)^{2}} \quad \Omega^{(1)} = \frac{1}{10} \quad \Omega^{(2)} = \frac{6}{10} \quad \Omega^{(3)} = \frac{3}{10} \quad (Shu, 1994)$$

Smoothness monitors

• Reconstruction of conservative variables

$$\mathbf{Q}_{ij}^{L} = \mathbf{S}_{ij} \mathcal{L}_{ij}^{L} = \mathbf{Q}_{i} + \mathbf{S}_{ij} \sum_{k=1}^{3} \omega_{k}^{L} SlopeL(k)$$

$$\mathbf{Q}_{ij}^{R} = \mathbf{S}_{ij} \mathcal{L}_{ij}^{R} = \mathbf{Q}_{j} - \mathbf{S}_{ij} \sum_{k=1}^{3} \omega_{k}^{R} SlopeR(k)$$

$$\mathbf{Riemann solver}$$

$$\mathbf{A}_{ij}^{R} = \mathbf{S}_{ij} \mathcal{L}_{ij}^{R} = \mathbf{Q}_{j} - \mathbf{S}_{ij} \sum_{k=1}^{3} \omega_{k}^{R} SlopeR(k)$$

$$\mathbf{A}_{ij}^{R} = \mathbf{S}_{ij} \mathcal{L}_{ij}^{R} = \mathbf{Q}_{j} - \mathbf{S}_{ij} \sum_{k=1}^{3} \omega_{k}^{R} SlopeR(k)$$

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$$\mathbf{A}_{ij}^{R} = \mathbf{A}_{ij} \sum_{k=1}^{3} (\mathbf{A}_{ij}^{R} - \mathbf{A}_{ij}^{R})$$

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Characteristic Quasi-FD WENO approach => WENO-EBR Scheme

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In-house code NOISEtte

Meshes

- unstructured meshes;
- barycentric (median) and orthocentric (CCS) control volumes

Basic mathematical model – full compressible Navier-Stokes equations

- 1eq Spalart-Allmaras turbulence model
- 2eq k-ε, k-ω, Menter SST

Approaches:

- Direct Numerical Simulation (DNS)
- (U)RANS, PANS
- LES (Smagorinsky, WALE, σ, ...)
- Nonzonal hybrid RANS-LES: (I)DDES

Numerical methods:

- space discretization: vertex-centered higher-accuracy EBR schemes (up to 6th),
- WENO/TVD-EBR techniques
- time integration: explicit Runge-Kutta 3-4 order scheme and implicit 1-2 order scheme (BiCGSTAB solver for SLAE)

High efficiency MPI+OpenMP parallelization

- large scale computations using meshes up to 10⁹ cells and up to 10⁵ CPUs
- parallel tools for pre and post processing

Far field acoustics: FWH method

Unstructured mesh

Speed-up of MPI+OpenMP parallelization

Mesh: 16M nodes, 100M tetrahedra

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Verification

- Algorithms realized in NOISEtte are verified on a large amount of test cases
- Results of verification on canonical turbulent flows will be presented here

Test case #1: Decay of isotropic homogenous turbulence

Initial field: generated u,v,w from E(k) at T=0

Calibration of C_{DES} constant:

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Verification

v-velocity

0.22

0.12
0.09
0.07
0.04
0.01
0.01
0.04

0.07

0.09

1.64

0.71
0.60
0.68
0.38

0.27

0.15

Test case #2: Turbulent planar channel flow

Objective: verification of WMLES branch of IDDES

Comparison: DNS by Jimenez et. al

Flow parameters:
$$\operatorname{Re}_{\tau} = 0.5 \cdot H \cdot u_{\tau} / \nu = 400$$

 $\operatorname{Re}_{b} = 1.4 \cdot 10^{4}$

Periodic on X and Z direction

Source term x pressure gradient to keep bulk velocity

Initial fields for v and w velocities, u=1 in entire domain

0.25 0.22 0.20 0.17 0.14

0.12

0.09 0.07 0.04 0.01 0.01 -0.01 -0.04

0.09

SST IDDES

SST IDDES 8.15 0.12 6.09 0.05 6.00 -0.03 4.09 4,12 SST IDDES 0.10 6.12 0.08 0.04 6.00 -0.04 -0.01 4.42 2.54 2.28 SST IDDES 2.28 2.07 1.76 1.50 1.24 0.98 0.72 0.46

w-velocity

Test case #3: Turbulent flow in channel with back facing step

Objective: to test DDES approach

Flow parameters: Re=28000 (H=3.8 cm, U₀=11.3 m/c (M=0.033)

Experimental data: Vogel & Eaton 1985

Computational domain:

Spanwise (Z) direction: periodicity, L_z=2H

Mesh: 0.74M nodes, 4.2M tetrahedrons. $\Delta z = H / 10$

Input BC: fixed velocity u and turbulent viscosity profiles (based on Θ =0.123H)

Numerical setup:

- Enhanced SA DDES (Strelets et al., 2014);
- Hybrid CD/UPW* method (Travin et al., 2004) based on EBR higher accuracy scheme

*pure CD scheme is unstable

Test case #3: Turbulent flow in channel with back facing step

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Simulations

Problem #1 Transonic flow over the wedge-shaped body with backward-facing step

Simulations: Problem #1

Experimental setup (by TsNIIMash)

Flow parameters: $Re_L = 7.2 \cdot 10^6$ $M_{\infty} = 0.913$ L = 0.358 [m]

Flow peculiarities:

- transonic flow self-rebuiling: flow structure rebuilding, increasing of OASPL
- interfering separation-induced flows in the base region

Objectives:

- verification of numerical methods;
- Investigation of the peculiarities and the physics of the flow

Experimental data:

- shadowgraphs;
- streamtraces on surfaces;
- profies of C_p OASPL PSD(p')

Simulations: Problem #1

Computational setup

- Approach: SA RANS, SA IDDES;
- Numerical scheme: hybrid EBR WENO-CD/UPW

Red: WENO-EBR Blue-green: CD/UPW* $\sigma_{upw}^{max} = 0.5$ in the area of interest

- Time integration: implicit 2nd order scheme. CFL<1 in the (LES) region of interest
- Mesh: 26M nodes, 149M tetrahedrons, spanwise resolution: $\Delta z = 2.10^{-3} \cong \delta / 6$
- Modification of LES sub-grid scale:

Visualization: turbulent structures

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Visualization: acoustics

C AA

Verification: mean flow

Visually recirculation zone is predicted correctly

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Simulations: Problem #1

Verification: mean flow profiles

Acceptable agreement with experiment on average pressure fluctuations level The difference can be explained by contribution of small-scales high-frequency turbulent pulsations considered at experimental levels and are not resolved in the simulation

Problem #2 Flow near tandem of square cylinders

Objective: investigation of noise generation mechanisms near landing gears

- EU FP7 VALIANT project
- Experiment: NLR, Netherlands
- Computational setup: SA DDES, EBR scheme
- Mesh: 12.5M nodes, 73M tetrahedrons

Problem #3

Numerical prediction of aerodynamic and aeroacoustic characteristics of axisymmetric flow generated by shrouded tail rotor

Objective: to evaluate aerodynamic and acoustic characteristics of ducted rotor for various rotor angular velocity and various pitch angles of rotor blades using inviscid-flow approach

Problem formulation

	1 mode	2 mode
Rotation frequency	f = 46.6 Hz	$f \approx 40.926 \mathrm{Hz}$
Angular velocity	$\omega = 292.80 \text{ rad/s}$	$\omega \approx 257.14 \text{ rad/s}$
Blade tip velocity	$V_{tip} \approx 205 \text{ m/s}$	$V_{tip} = 180 \text{ m/s}$

Simulations: Problem #3

Geometry, Computational domain, Boundary Surfaces

Mathematical model:

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Euler equations 
in a non-inertial rotating frame of reference
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 $\mathbf{V} = \mathbf{\omega} \times \mathbf{r}$ $\frac{\partial \rho}{\partial t} + \operatorname{div} \rho (\mathbf{u} - \mathbf{V}) = 0$ $\frac{\partial \rho \mathbf{u}}{\partial t} + \operatorname{Div} \rho (\mathbf{u} - \mathbf{V}) \otimes \mathbf{u} + \nabla p = -\rho (\mathbf{\omega} \times \mathbf{u})$ $\frac{\partial E}{\partial t} + \operatorname{div} (\mathbf{u} - \mathbf{V}) E + \operatorname{div} \mathbf{u} p = 0$

Mesh:

Number of nodes:

2.3-2.7 млн

Number of tetrahedra: 13.6-15.2 млн

Numerical results: flow pattern

Streamtraces colered by velocity magnitude

Aerodynamic forces

Rotor, ring and total thrust coefficients

Rotor, ring torque coefficient

There is no experimental data. The correctness of the simulations is confirmed by the selfsimilarity of solution at different blade tip velocities.

Far field pressure pulsations

FW-H: Directivity diagram depending on control surface

Overall Sound Pressure Level (OASPL) at the distance 150m

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Thank you for attention!