Numerical investigation of nonaxisymmetric instability growth in the 3D bubble-shock interaction problem

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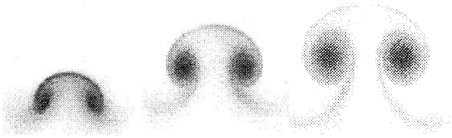
Bubble-shock interaction problem

This problem is linked to the important tasks such as

1. Description of turbulent combustion in the jet engines
2. Interaction of fuel slurry with a shock wave of the piston in the internal combustion engines
3. Sonoluminescence
4. Non-surgical removal of kidney stones (lithotripsy)

First experimental results appeared in 1960s (Rudinger et al.)



Since 1990 various works with the numerical simulation of the problem have been produced.

Euler equations of fluid dynamics

In this work Euler equation of the dynamics of inviscid fluid in 3D is used

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial z} = 0, \text{ where}$$

$\mathbf{U} = (\rho, \rho u, \rho v, \rho w, E)^T$ is a vector of conservative variables,
$E = \rho(\varepsilon + \frac{1}{2}(u^2 + v^2 + w^2))$ is full energy
$\mathbf{F}(\mathbf{U}) = (\rho u, \rho u^2 + p, \rho uv, \rho uw, (E + p)u)^T$,
$\mathbf{G}(\mathbf{U}) = (\rho v, \rho vu, \rho v^2 + p, \rho wv, (E + p)v)^T$,
$\mathbf{H}(\mathbf{U}) = (\rho w, \rho uw, \rho vw, \rho w^2 + p, (E + p)w)^T$ are eulerian fluxes,
$p = p(\rho, \varepsilon)$ is equation of state which completes the system of equations

EoS of the ideal gas is used, so $p = \rho\varepsilon(\gamma - 1)$.

Runge-Kutta discontinuous Galerkin (RKDG) method

Approximate solution U_h at every cell L_j is written in form of

$$\mathbf{U_h} = \sum_{i=1}^{k} \mathbf{U}_i(t)\varphi_i(x), \text{ where } \{\varphi_i(x)\}_{i=1}^{k} \text{are specified basic functions}$$

$$\mathbf{U}_i(t) = [u_i^1(t), \dots, u_i^n(t)]^T \text{ time-dependent vector, } n = 5 \text{ for 3D flow}$$

For every component we get

$$u_h^s = \sum_{i=1}^{k} u_i^s(t)\varphi_i(x);\ s = 1, \dots, 5$$

Then inserting this approximate solution $\mathbf{U}_i(t)$ for every $L_j,\ i = \overline{1,k},\ l = \overline{1,k},\ j = \overline{1,N},\ m = 1,\ldots,5$ and using the Divergence theorem such a system is obtained

$$\sum_{i=1}^{k} \frac{\partial u_i^m(t)}{\partial t} \int\limits_{L_j} \varphi_i \varphi_l dV - \int\limits_{L_j} (\mathbf{F}^m \frac{\partial \varphi_l}{\partial x} + \mathbf{G}^m \frac{\partial \varphi_l}{\partial y} + \mathbf{H}^m \frac{\partial \varphi_l}{\partial z}) dV + \\ + \int\limits_{\delta L_j} \varphi_l (\tilde{\mathbf{F}}^m dydz + \tilde{\mathbf{G}}^m dzdx + \tilde{\mathbf{H}}^m dxdy) = 0,$$

where $\tilde{\mathbf{F}},\ \tilde{\mathbf{G}},\ \tilde{\mathbf{F}}$ are are Godunov-type numerical fluxes

Runge-Kutta with the limiter

Considered ODE system can be represented in the form of

$$\frac{du(t)}{dt} = Lu(t);$$
$$u(0) = u_0;$$

In terms of the RKDG method the explicit Runge-Kutta method is supposed to apply

- limiting the solution in a special way after each RK stage

It is necessary to use the limiter for suppressing the spurious oscillations near big gradients.

- Various types of limiters have been developed (minmod, WENO etc)
- Reconstruction of the coefficients is a local procedure

The results of the constructing

The method has the following features

- Explicit with local stencil
- High-order accuracy in time and space
- Non-oscillating

Specifications in this work

- 3D Cartesian grid
- 2^{nd} order in space and time RKDG method (with piecewise linear elements)
- Harten-Lax-van Leer-Contact (HLLC) numerical flux
- Minmod limiter is used (TVDM)

Impact of LRnLA algoritms using

LRnLA stands for "Locally Recursive non-Locally Asynchronous"

Locality Take advantage of memory subsystem hierarchy, from on-chip CPU cash and up to disk and network

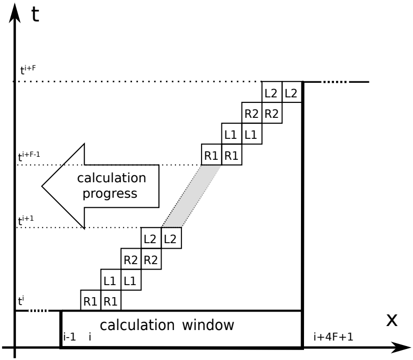
Recursivity Application of "divide et impera" strategy for any situations (computer architectures, numerical schemes, etc.)

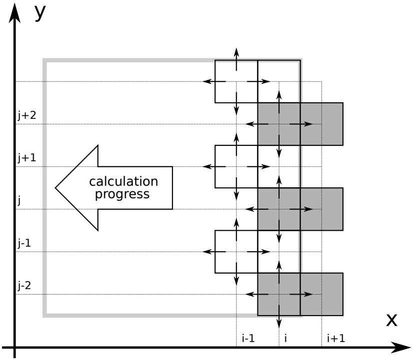
non-Locality Optimized for distributed computations

Asynchrony Adaptable parallel computations on any levels

- Structure is well-compatible with the GPU architecture
- Effective GPU-CPU interaction
- Close to the maximum performance

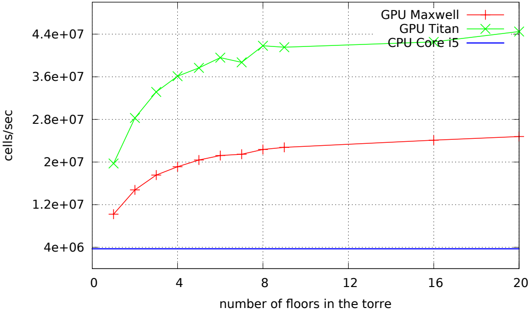
Scheme of the domain calculation by the DiamondTorre algorithm





- "Torres" are vectorized by z axis
- "Torres" are calculated asynchronously by CUDA-blocks
- Calculations inside a «window» of GPU memory, while the entire domain is in CPU memory

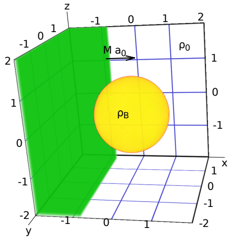
Performance results



- Performance gained on a single GPU NVidia GTX Titan is about $4.5 \cdot 10^7$ cells per second
- Using 32GB CPU DDR the task with the domain of about $4 \cdot 10^8$ cells can be calculated

Units of measure

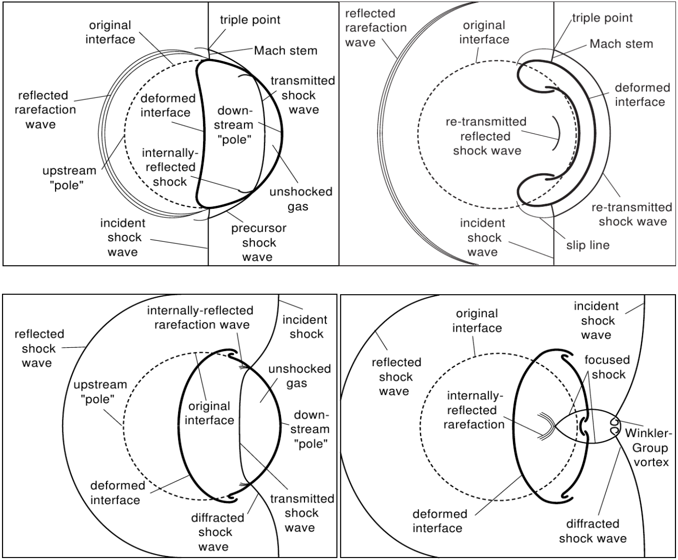
The dimensionless units are chosen connected with the radius of a bubble, the background sound velocity and density by the following expressions



$$x = R_0\tilde{x},\ y = R_0\tilde{y},\ z = R_0\tilde{z};$$
$$u = a_0\tilde{u},\ v = a_0\tilde{v},\ w = a_0\tilde{w};$$
$$t = \tilde{t}R_0/a_0;$$
$$\rho = \rho_0\tilde{\rho};$$
$$p = \rho_0 a_0^2\tilde{p} = \tilde{p}p_0/\gamma.$$

The shock wave is characterized by the Mach number M of its front moving velocity. The Atwood number $At = \frac{\rho_B - \rho_0}{\rho_B + \rho_0}$ is used for the characteristic of the density inside the bubble.

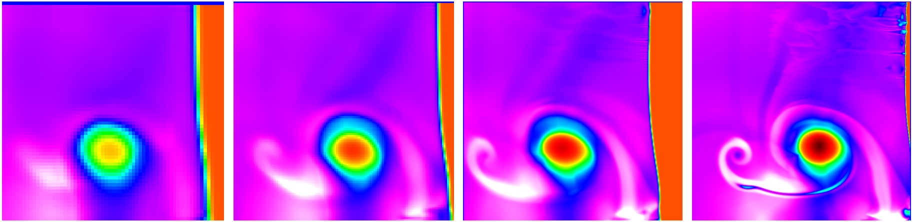
Schemes of the process at $At < 0$ (top) and $At > 0$ (bottom)



Niederhaus J. H. J. et al. A computational parameter study for the three-dimensional shock-bubble interaction //Journal of Fluid Mechanics. – 2008. – V. 594. – P. 85-124.

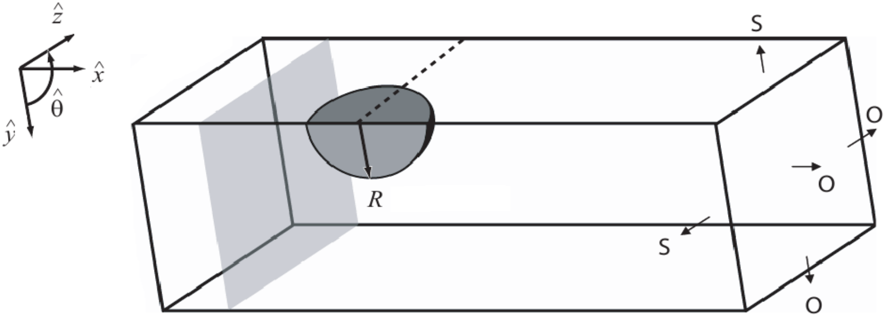
Mesh-sensitive effects

The same task has been solved on several coarse or more fine grids. Radius of a bubble contains a) 32, b) 64, c) 128, d) 256 cells

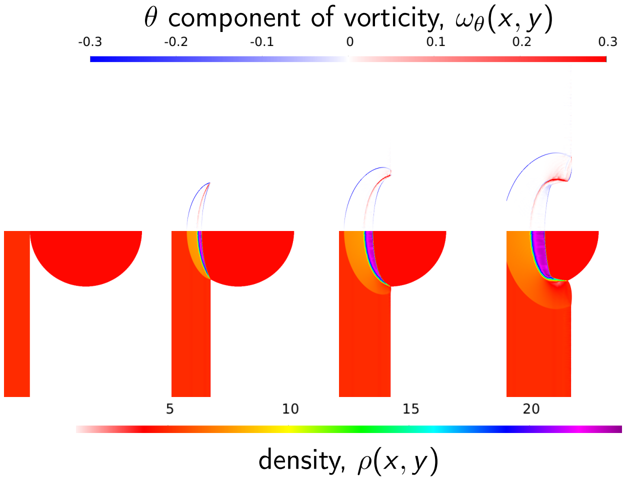


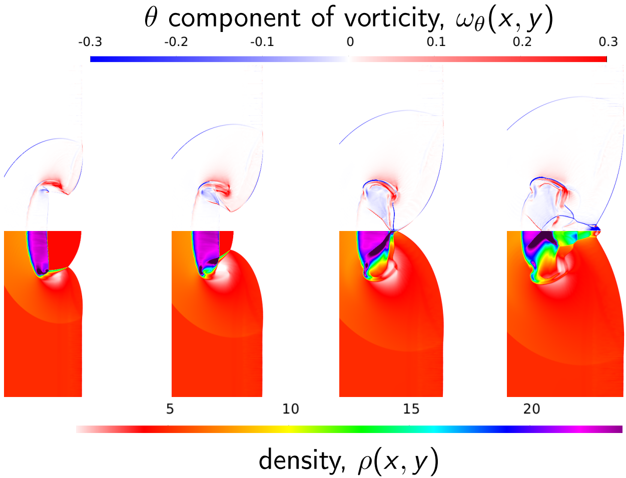
(a) (b) (c) (d)

Numerical domain

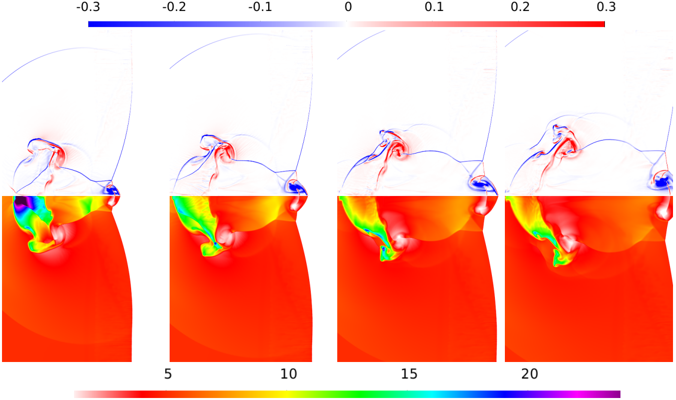


Due to the symmetry one quarter of the whole domain can be considered, introducing the boundary condition of symmetry on the diametrical cross-sections.

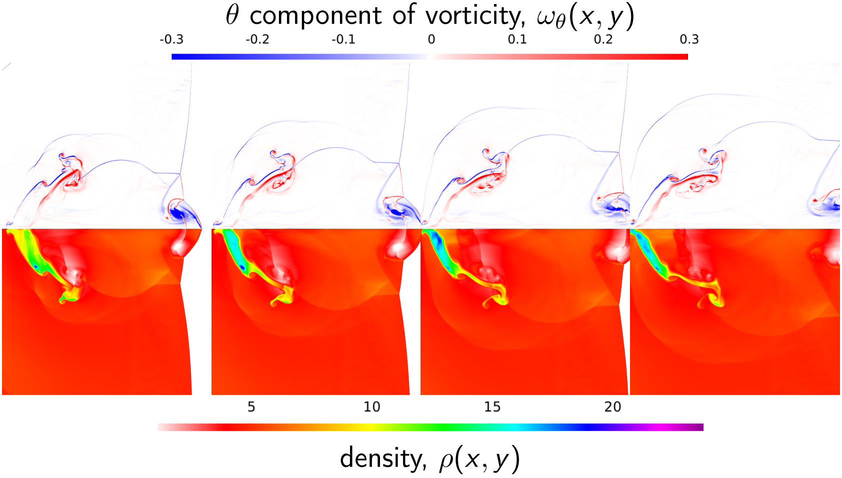

θ component of vorticity, $\omega_\theta(x, y)$
-0.3 -0.2 -0.1 0 0.1 0.2 0.3
5 10 15 20
density, $\rho(x, y)$

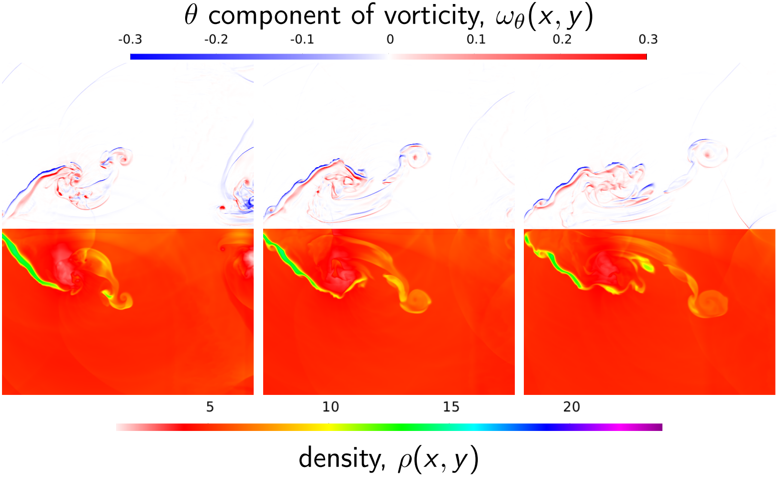

θ component of vorticity, $\omega_\theta(x, y)$
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
5
10
15
20
density, $\rho(x, y)$

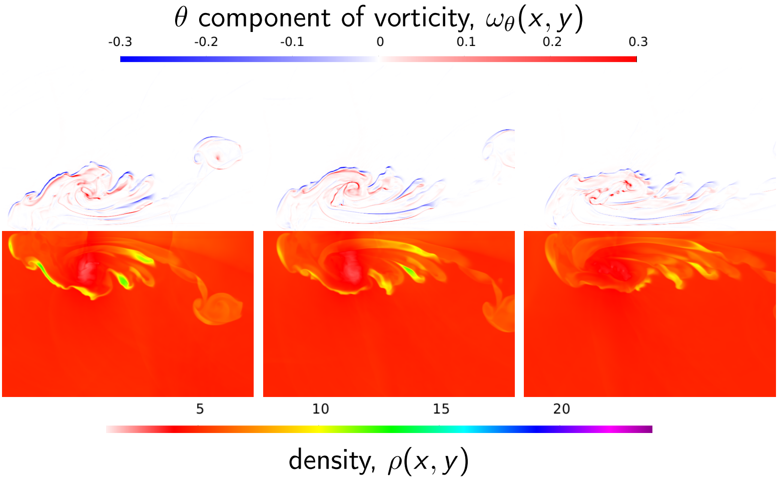
θ component of vorticity, $\omega_\theta(x, y)$



density, $\rho(x, y)$

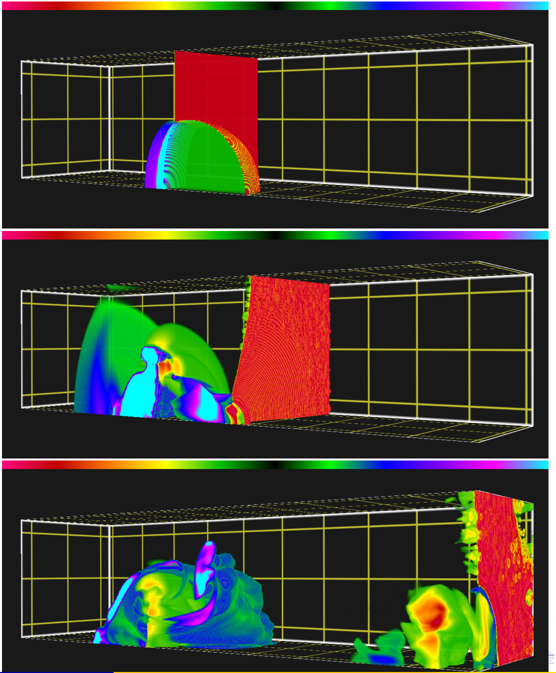

θ component of vorticity, $\omega_\theta(x, y)$
-0.3 -0.2 -0.1 0 0.1 0.2 0.3
5 10 15 20
density, $\rho(x, y)$


θ component of vorticity, $\omega_\theta(x, y)$
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
5
10
15
20
density, $\rho(x, y)$


θ component of vorticity, ω_θ(x, y)
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
5
10
15
20
density, ρ(x, y)

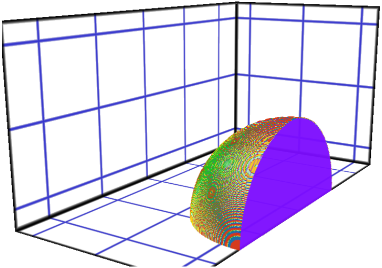
Turbulent character of the process

There should be a movie



The perturbation of the bubble surface is defined by

$$Y_m^m(\varphi,\theta) = \alpha \sin(m\varphi)\sin^m(\theta),\ \alpha \ll 1, \quad m = 4\dots128$$



$N_x \times N_y \times N_z = 512 \times 256 \times 256$ cells, time $0 \leq t \leq 2.4$ ($2 \cdot 10^4$ time steps) is about 3 hours of computer time on 1 node of K100

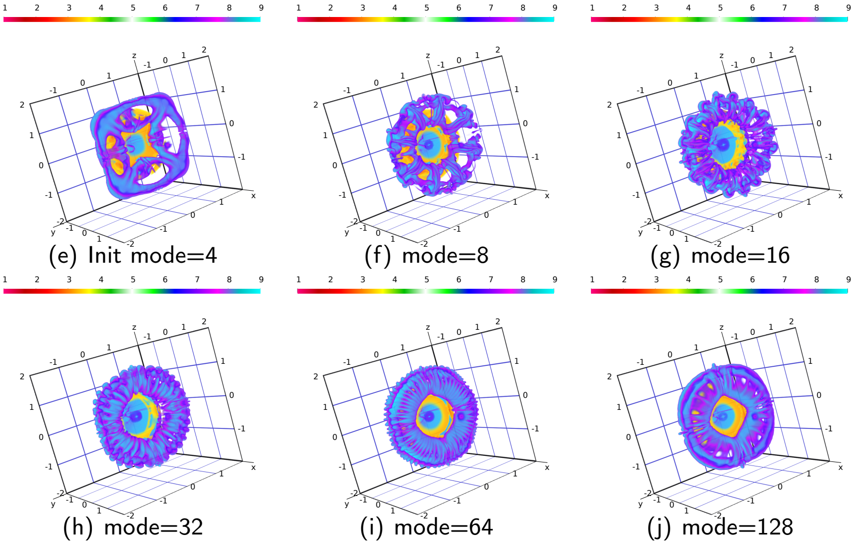
$$\rho(x,y,z,t) \rightarrow \rho_\theta(x,\theta,t) = \Sigma \rho r \Delta r / R_0$$

Spectral analysis:

$$\rightarrow \rho_k(x,t) = \left| \Sigma \rho_\theta e^{-ik\theta} \right|$$

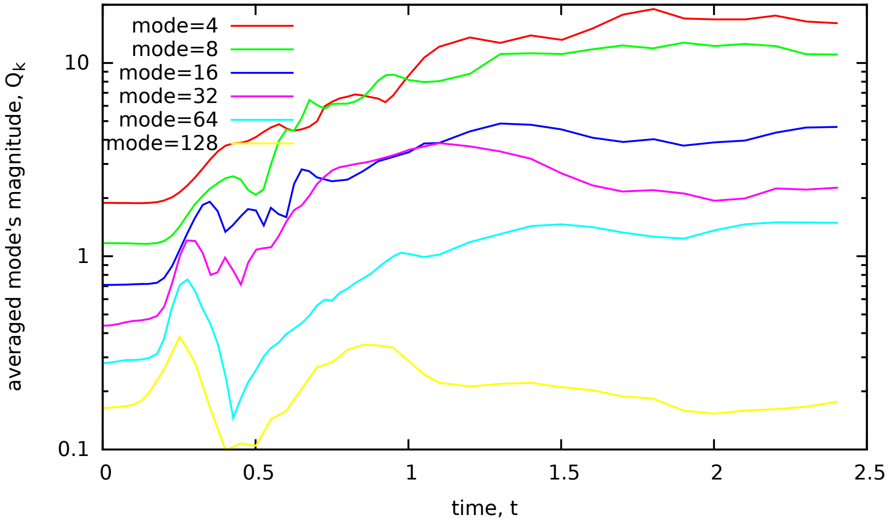
$$\rightarrow Q_k(t) = \Sigma \rho_k(x,t) \Delta x$$

Bubble's shape after shock wave passing (t=1.8)

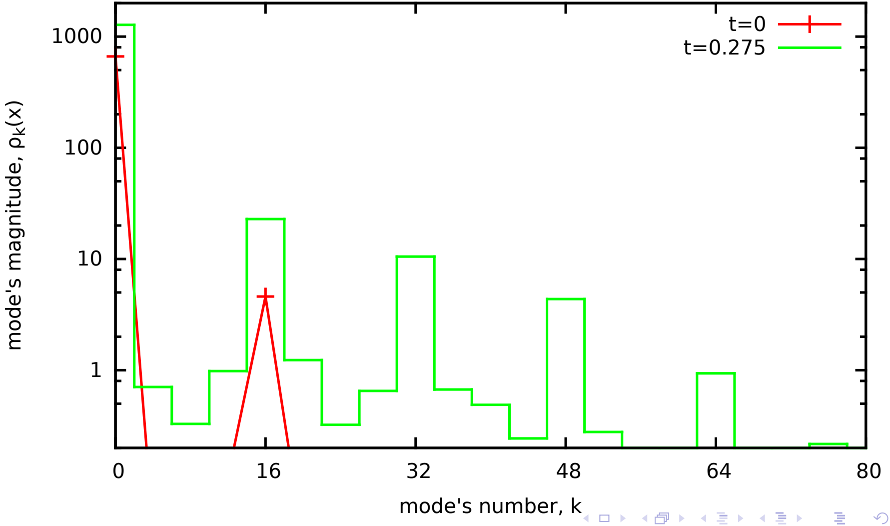


(e) Init mode=4
(f) mode=8
(g) mode=16
(h) mode=32
(i) mode=64
(j) mode=128

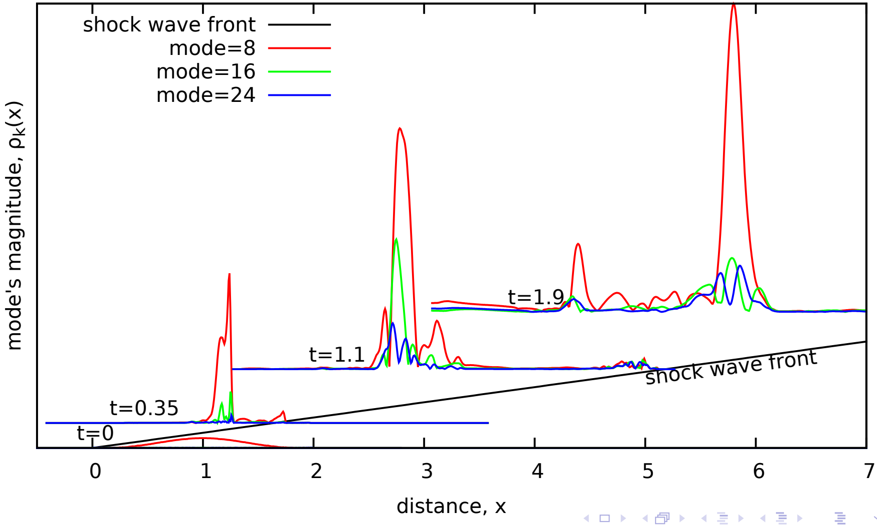
Evolution of the initially perturbated modes



Multiple modes' arising



Modes x-t dynamics



Summary

GPU solver for 3D CFD problems based on the RKDG method and the DiamondTorre LRnLA implementation algorithm is presented.

- Bubble-shock interaction problem is investigated in high-density regime and the axial non-stability is observed;
- Spectral analysis of density distribution denote some special aspects of the process as follows:
  - The initial perturbation is outstanding up to last stage,
  - on the initial stage increment grows with mode number,
  - multiple modes arise after shock wave passing throwg bubble.