APAN-RUSSIA WORKSHOP ON SUPERCOMPUTER MODELING, INSTABILITY AND TURBULENC
IN FLUID DYNAMICS (JR SMIT2015)
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SUPERCOMPUTER MATHEMATICAL MODELLING OF MATTER FLOWS IN ACCRETION STELLAR DISKS

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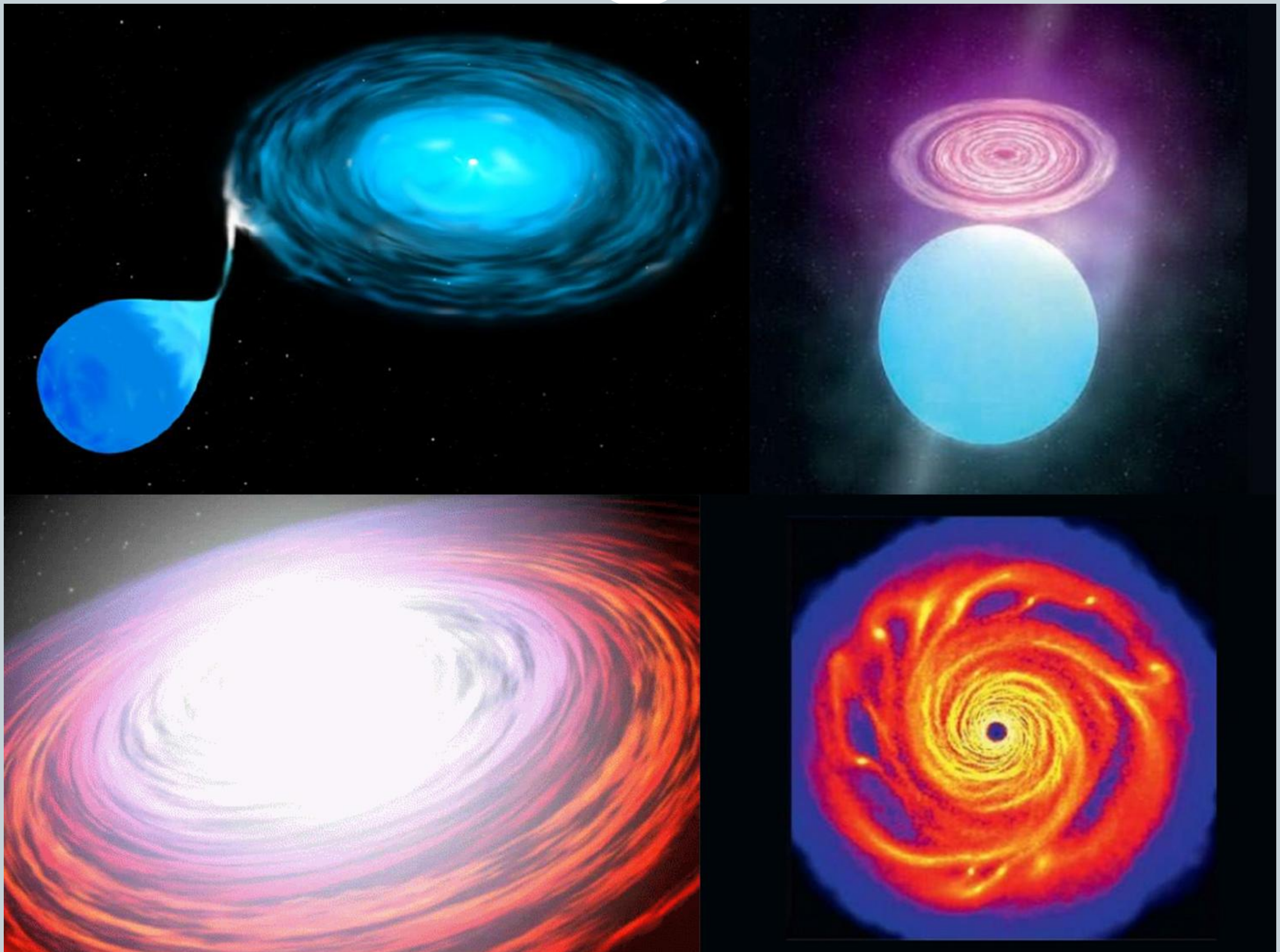
ACCRETION STELLAR DISK:
interpretation of observations and modeling results



MAIN IDEAS

1. In free shear flow of the matter of an accretion disk the hydrodynamical instability may arise.
2. This instability leads to the formation of the large-scale vortex flow.
3. Large–scale vortexes could transfer the angular momentum to the outer parts of the accretion stellar disk without heating of the matter.

ALSO

1. The comparison of the modeling results of the 2D and 3D approaches will be shown.
2. In MHD approach the arising of magnetorotational instability leads to the formation of the large-scale vortexes which could transfer the angular momentum to the outer parts of the accretion disk.
3. Some results of the modeling of the spiral galaxy structure received by using the mathematical model of the accretion disk are shown.

MAIN EQUATIONS

The flow of compressible ideal gas in gravitational field is described by the Euler equations of classical gas dynamics in cylindrical coordinates.

$$\frac{\partial(r\rho)}{\partial t}+\frac{\partial(r\rho u)}{\partial r}+\frac{1}{r}\frac{\partial(r\rho v)}{\partial \phi}+\frac{\partial(r\rho w)}{\partial z}=0$$

$$\frac{\partial(r\rho u)}{\partial t}+\frac{\partial(r\rho u^2+r\,p)}{\partial r}+\frac{1}{r}\frac{\partial(r\rho u v)}{\partial \varphi}+\frac{\partial(r\rho u w)}{\partial z}=p+\rho \mathrm{v}^2+r\rho F_{gr}$$

$$\frac{\partial(r\rho v)}{\partial t}+\frac{\partial(r\rho v u)}{\partial r}+\frac{1}{r}\frac{\partial(r\rho v^2+r\,p)}{\partial \varphi}+\frac{\partial(r\rho v w)}{\partial z}=-\rho u \mathrm{v}+r\rho F_{g\varphi}$$

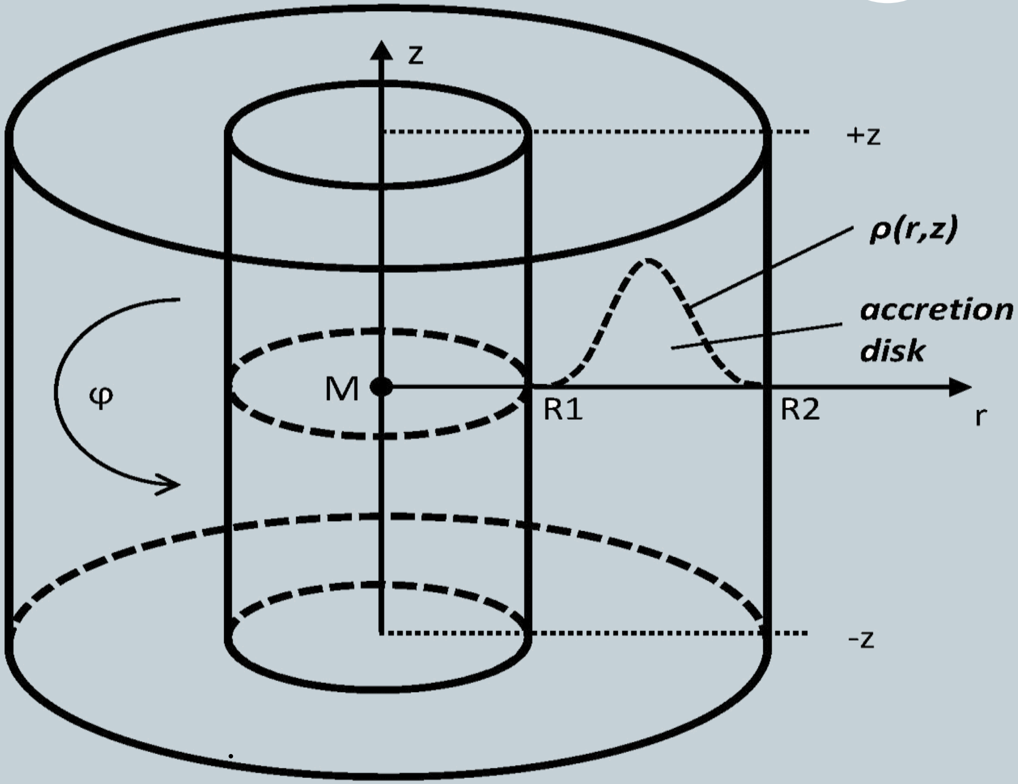
$$\frac{\partial(r\rho w)}{\partial t}+\frac{\partial(r\rho w u)}{\partial r}+\frac{1}{r}\frac{\partial(r\rho w v)}{\partial \varphi}+\frac{\partial(r\rho w^2+r\,p)}{\partial z}=r\rho F_{gz}$$

$$\frac{\partial(r\rho e)}{\partial t}+\frac{\partial(r\rho u h)}{\cdot\,\partial r}+\frac{1}{r}\frac{\partial(r\rho v h)}{\partial \varphi}+\frac{\partial(r\rho w h)}{\partial z}=r\rho(\mathbf{F_g},\mathbf{V}),$$

$$e=\varepsilon+\frac{\mathbf{V}^2}{2}=\frac{u^2}{2}+\frac{\mathrm{v}^2}{2}+\frac{w^2}{2},h=e+p/\rho.$$

The equation of state of an ideal gas: $p=(\gamma-1)\rho\varepsilon.$

FORMULATION OF THE PROBLEM



Numerical parameters:

$$M = 2 \bullet 10^{33} - 6 \bullet 10^{33}\, g$$

$$R = 10^{11} - 10^{14}\, cm$$

$$\rho_{\max} / \rho_{\min} = 10^5$$

«Free» boundary conditions:

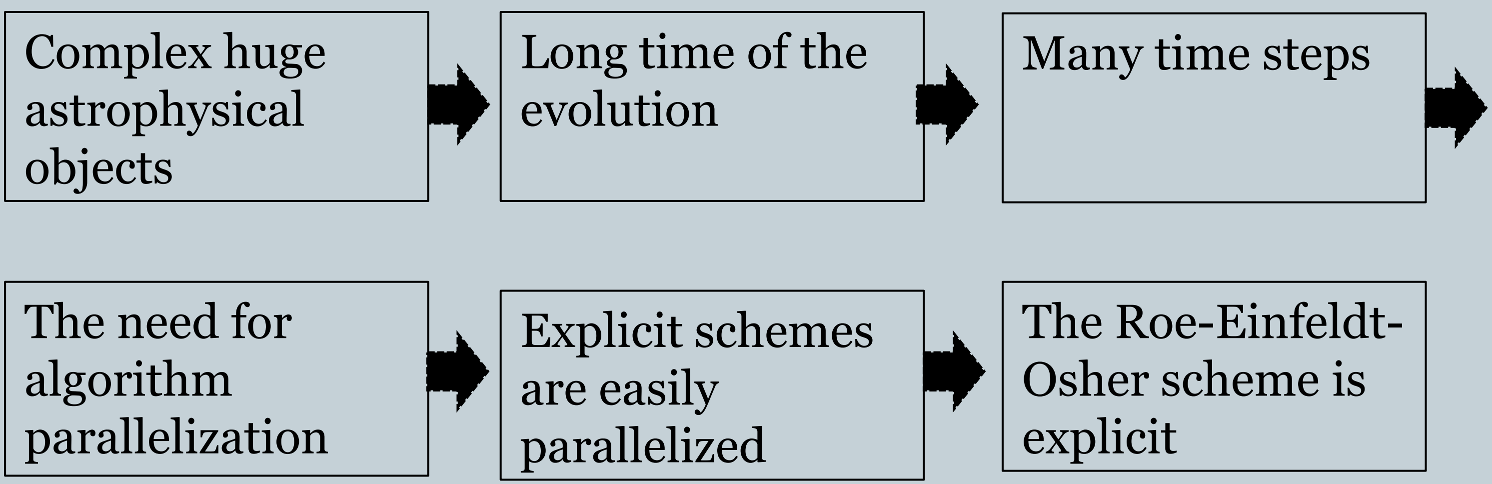
$$\left.\frac{\partial f}{\partial r}\right|_{r=R_1,R_2} = 0, \left.\frac{\partial f}{\partial z}\right|_{z=-Z,Z} = 0$$

$$f = \rho, u, \mathrm{v}, \mathrm{w}, e$$

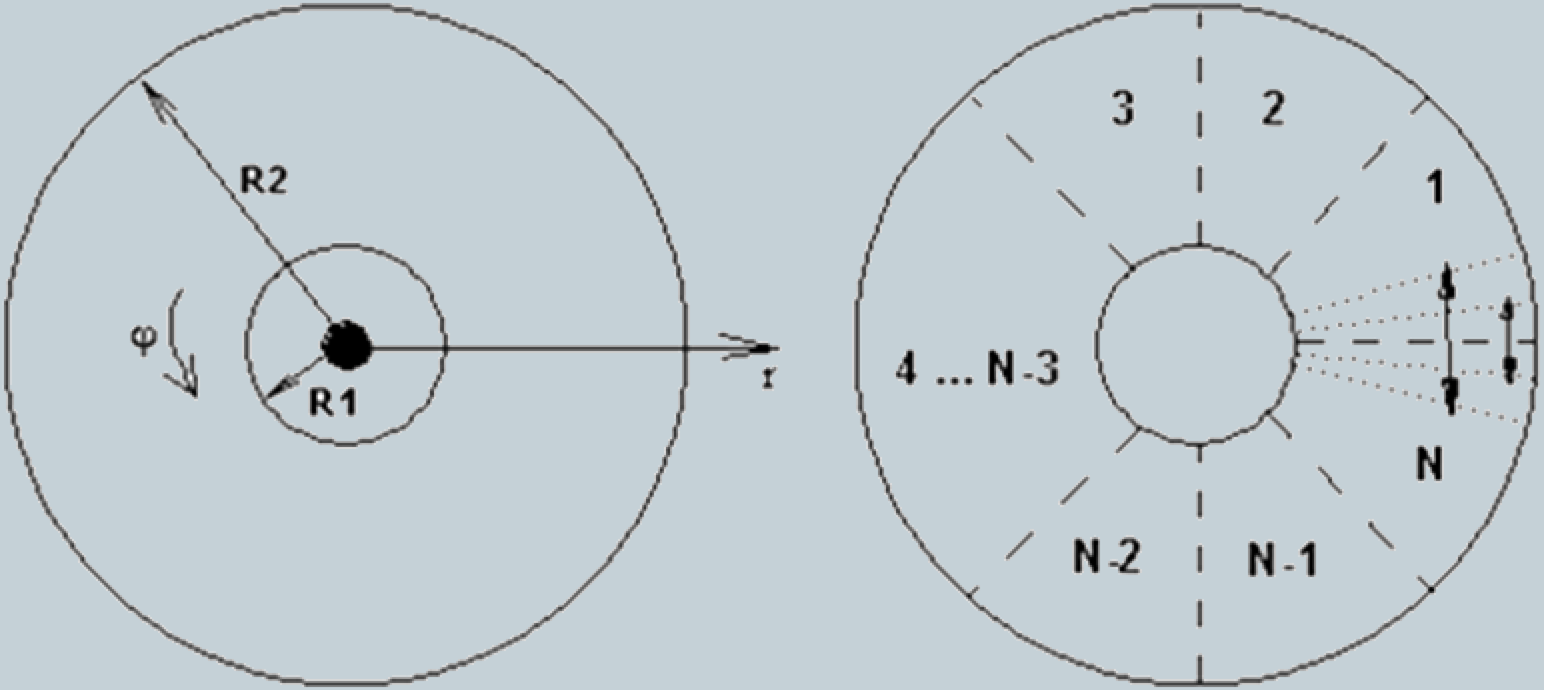
The initial conditions of the stationary equilibrium gas configuration are analytically specified using the technique proposed in [*M.V. Abakumov, S.I. Mukhin, Yu.P. Popov, and V.M. Chechetkin, Astron. Rep. 40, 366 (1996)*]

COMPUTATION METHOD

The problem is solved using the Godunov-type monotonic conservative ***Roe-Einfeldt-Osher scheme*** (TVD) with third order of approximation (due to limiters using). This scheme has minimum numerical dissipation in the class of such schemes.

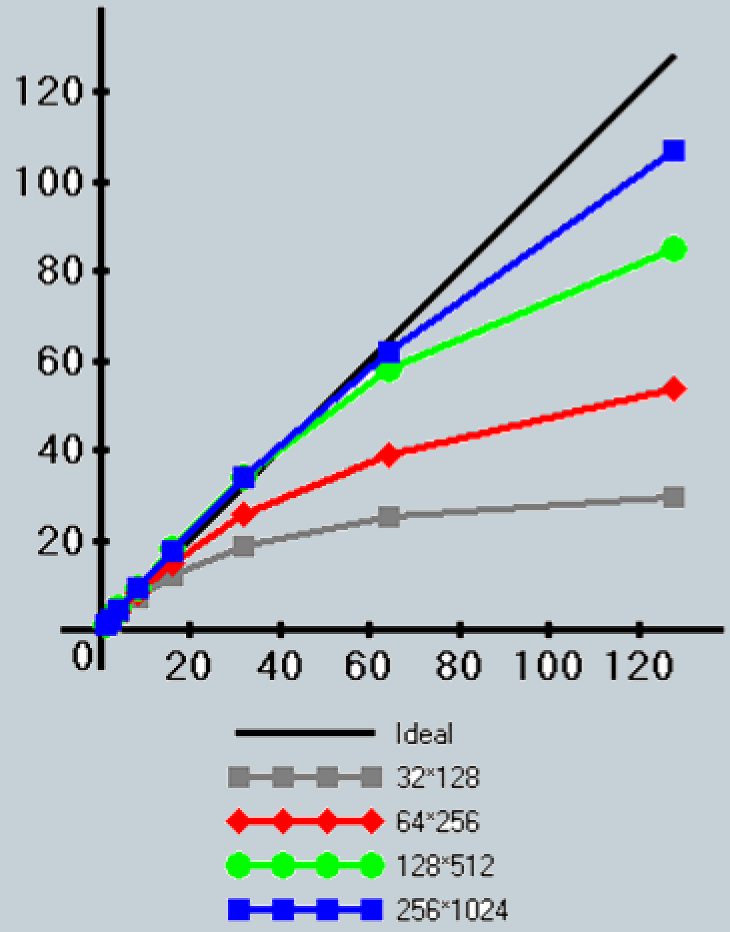


PARALLELIZATION TECHNIQUE

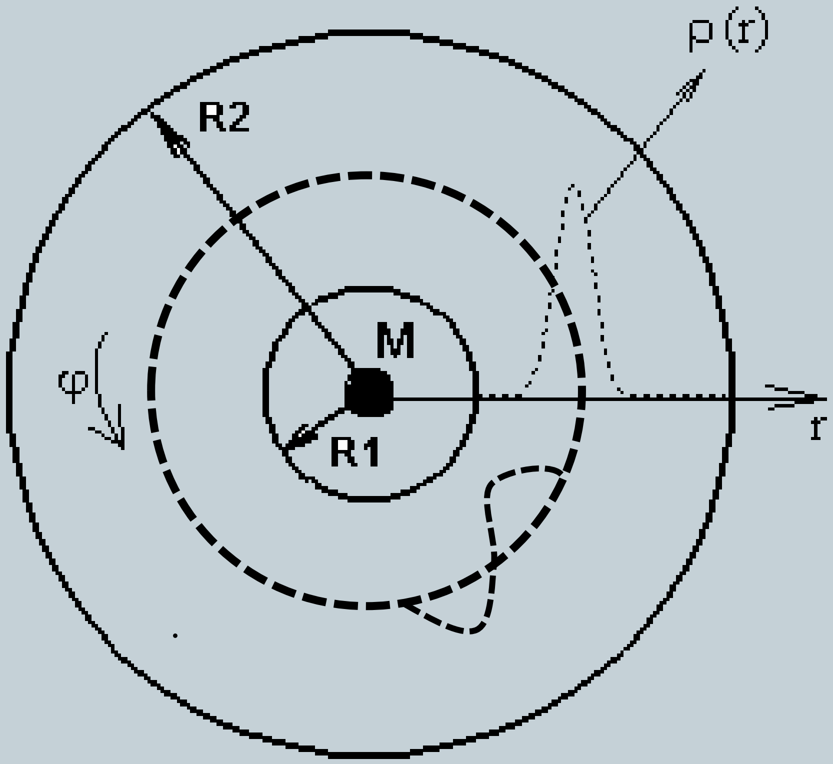


The principle of domain decomposition and
using of library functions MPI (Message Passing Interface)

PARALLELIZATION EFFICIENCY



2D MODELING



$$V(r,\varphi) = V_{равн}(r)\left[1 + A\sin(n\varphi)\right]$$

$$0 \le \varphi \le 2\pi$$

$$A = 0.2, n = 10$$

Note: A = 0.2 corresponds to the kinetic energy perturbation at 0.1%

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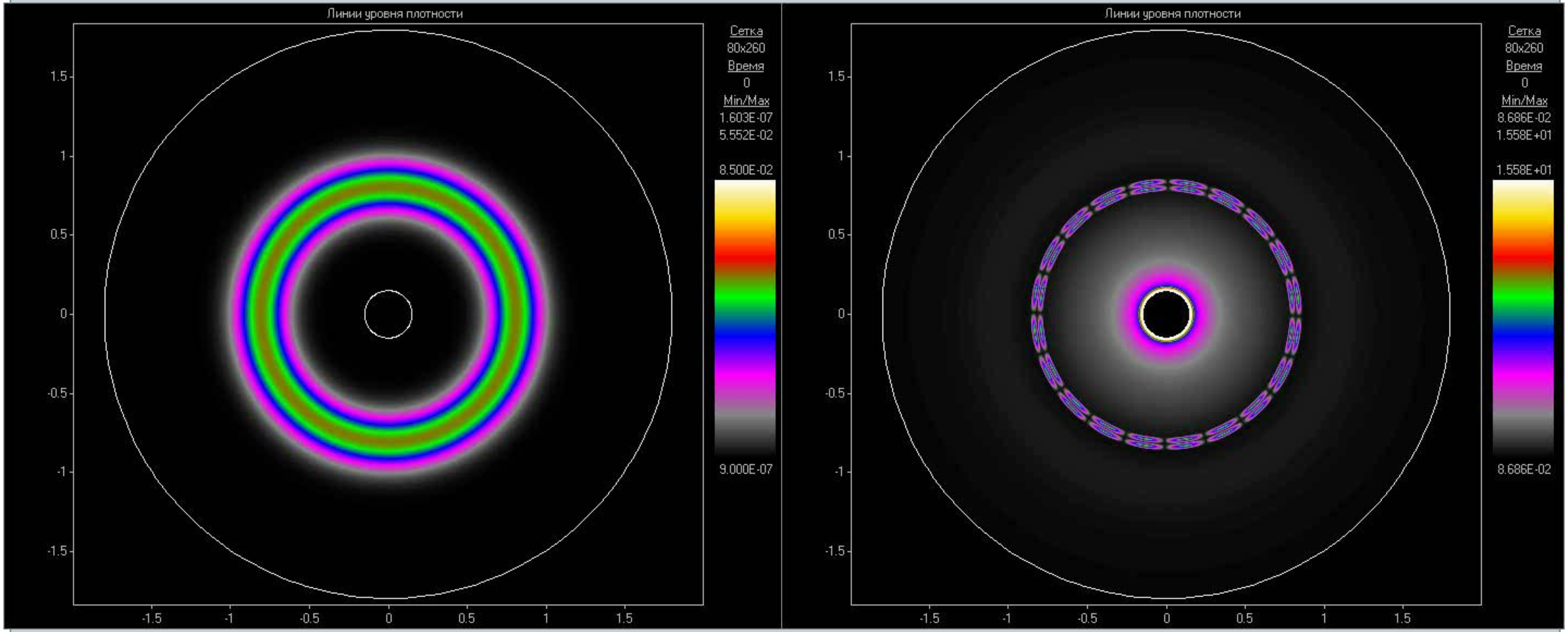
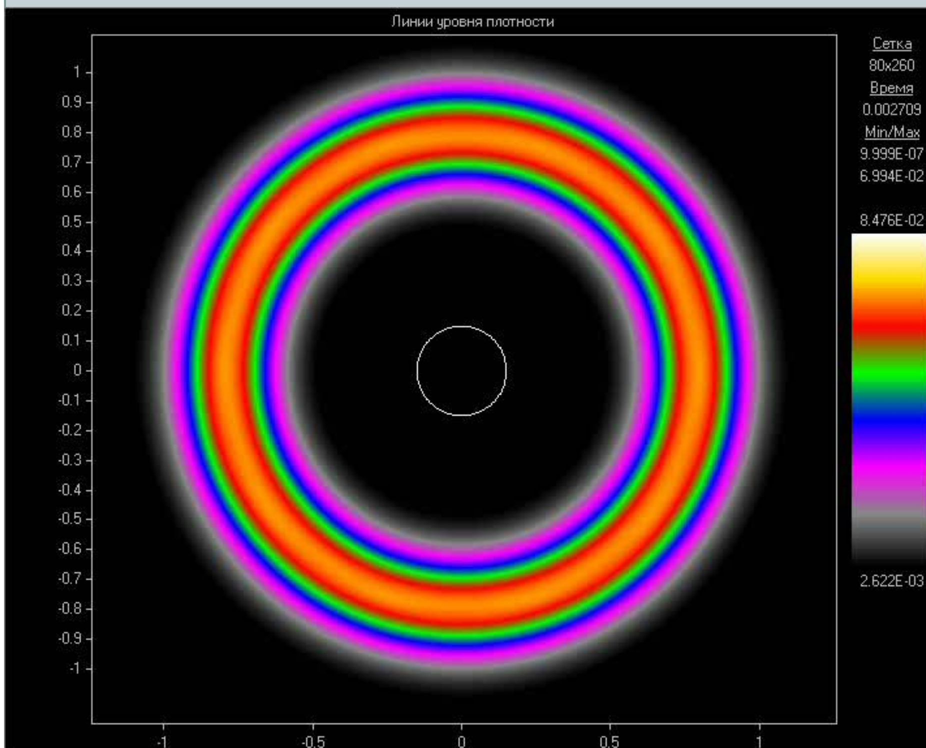
FORMATION AND EVOLUTION OF VORTEX FLOWS



Density (ρ)

Vorticity (|rotV|)

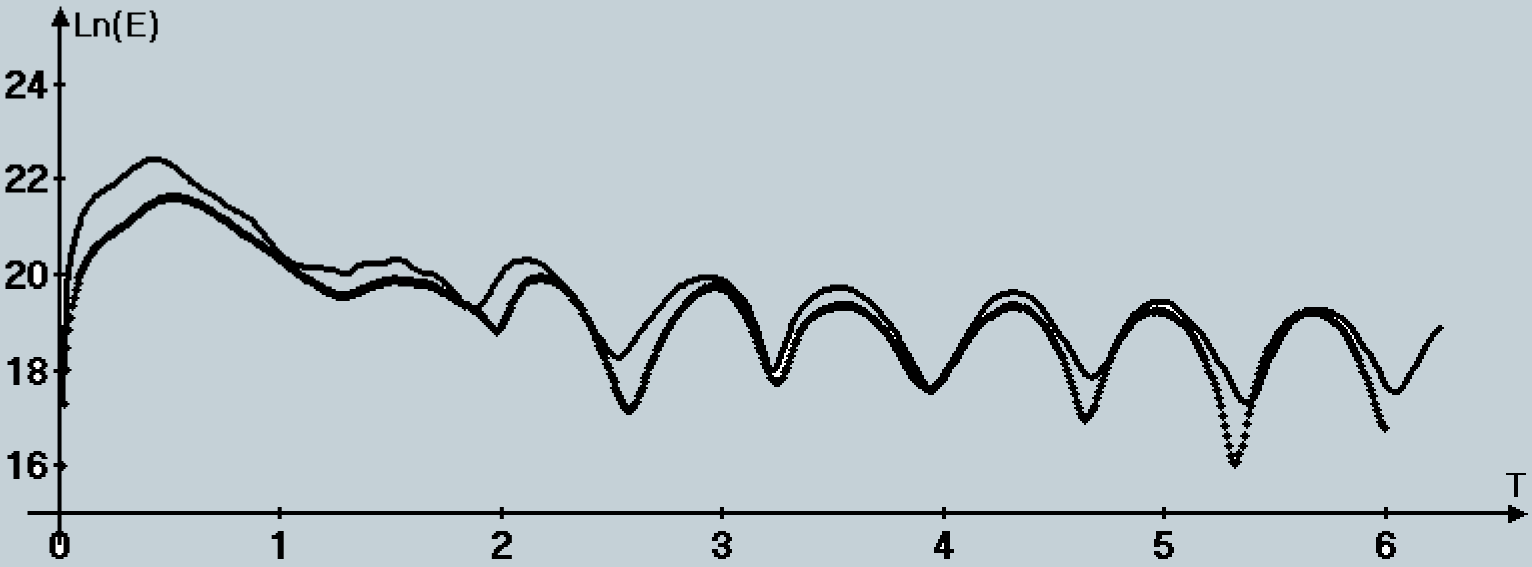
REDISTRIBUTION OF ANGULAR MOMENTUM



Distribution of density in time

Distribution of angular momentum along the radius (blue – $\varphi = 0$)

VARIATIONS OF THE KINETIC ENERGY IN VORTEX MOTION



Time in revolutions

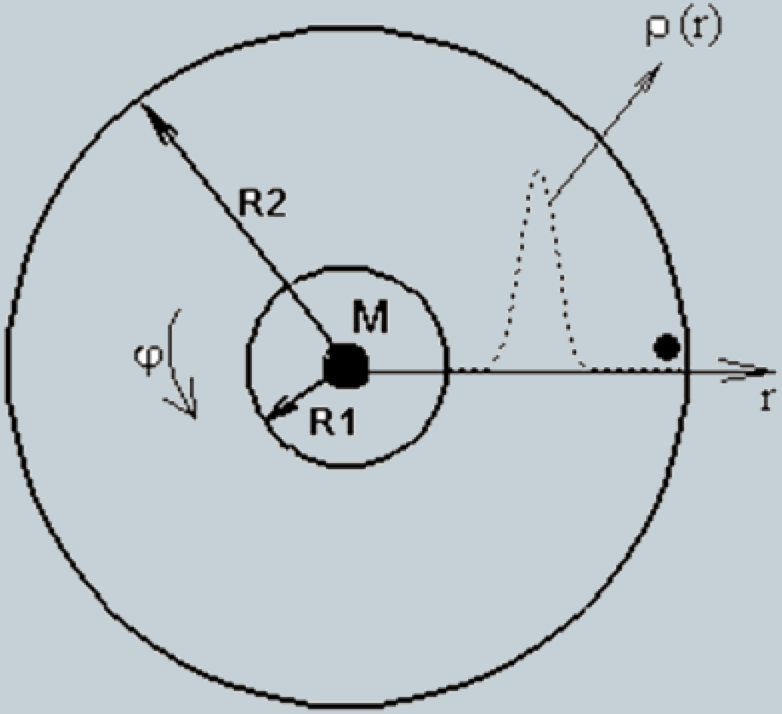
A = 0.2 (▬▬▬) и A = 0.1 (●●●)

ANGULAR MOMENTUM TRANSFER WITHOUT HEATING

1. Large–scale vortexes could transfer the angular momentum to the outer parts of the accretion stellar disk.
2. We show that the entropy in the system is constant.
3. We show that the total mass and total angular momentum don’t change during the evolution of the system.

New mechanism of angular momentum transfer by large vortex structures without significant heating of the disk matter.

DISTURBANCE IN THE OUTER PART OF AN ACCRETION DISK

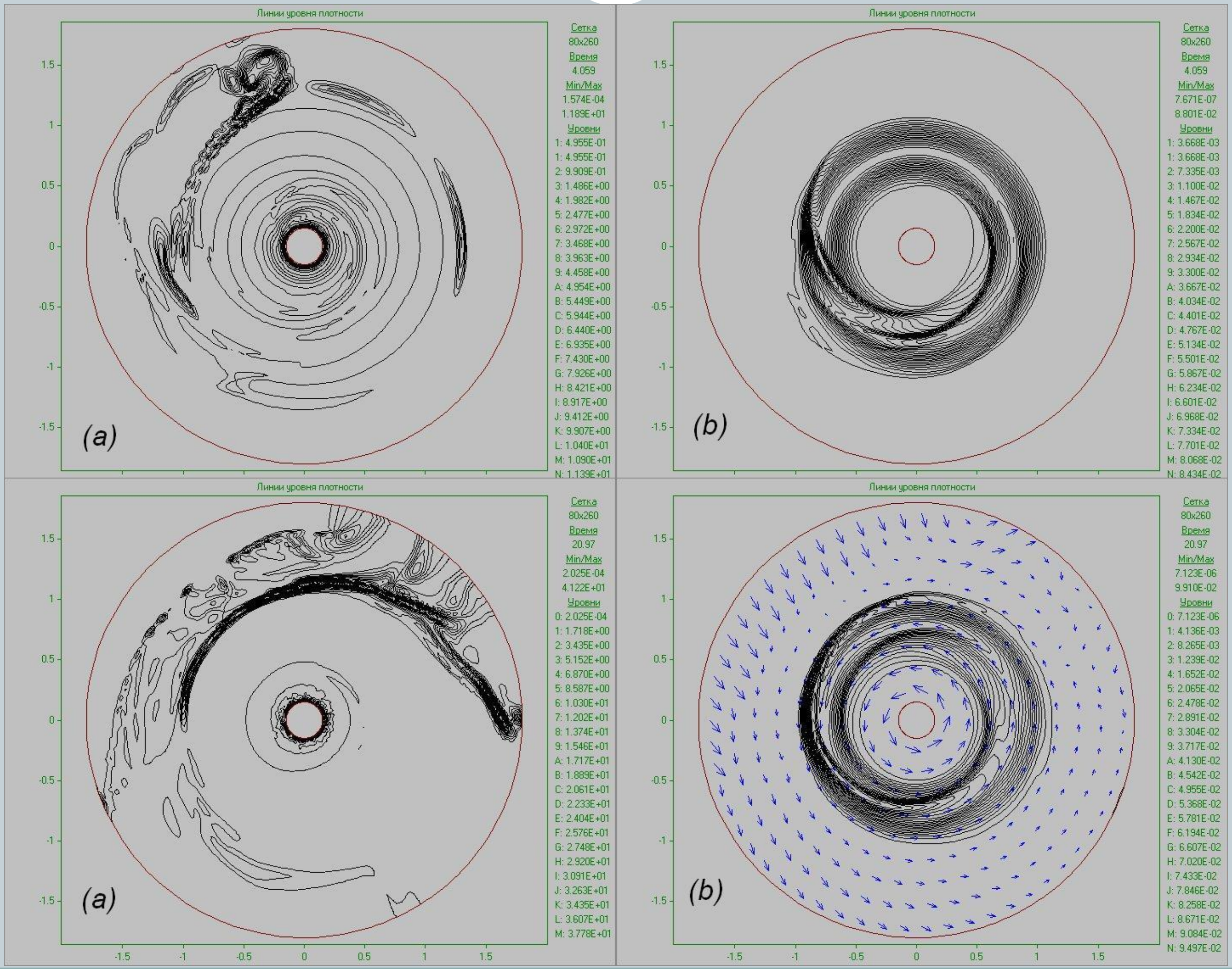


$$v(r,\varphi) = a v_{равн}(r), r_1 \le r \le r_2, \varphi_1 \le \varphi \le \varphi_2, a < 1;$$

$$\rho(r,\varphi) = b \rho_{равн}(r), r_1 \le r \le r_2, \varphi_1 \le \varphi \le \varphi_2, b > 1.$$

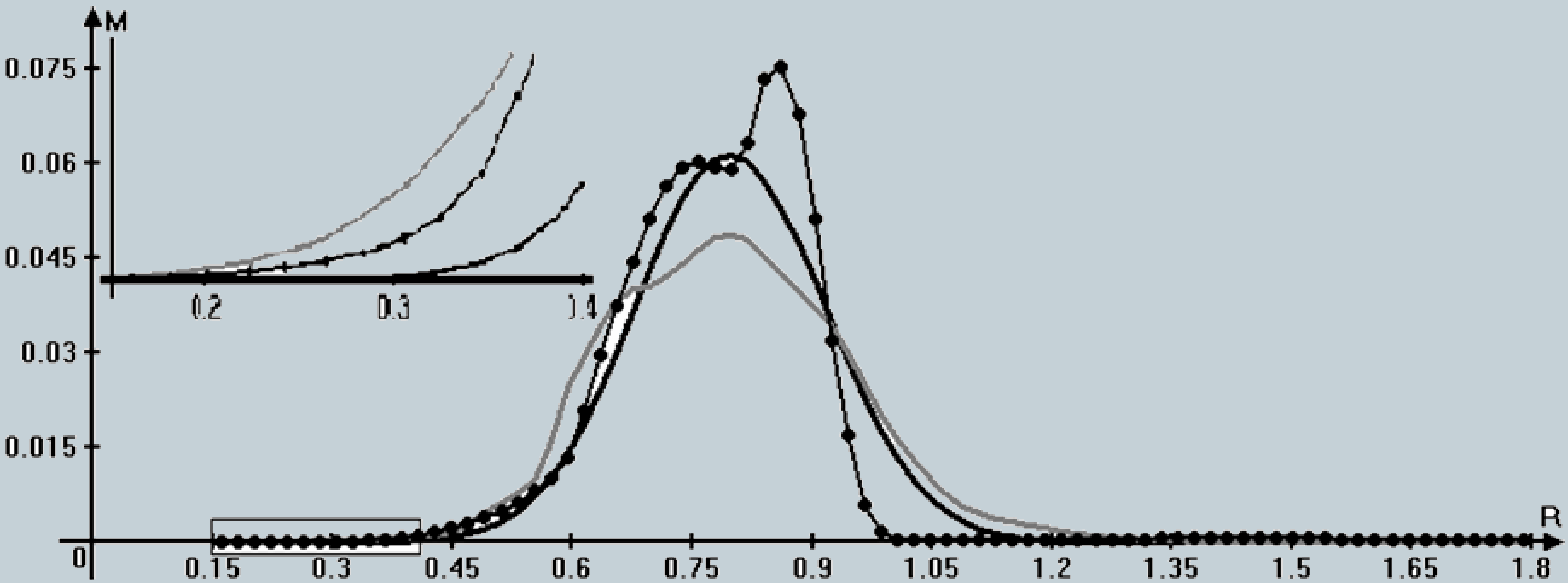
LOCAL DISTURBANCE

Vorticity
(rot V)



Density

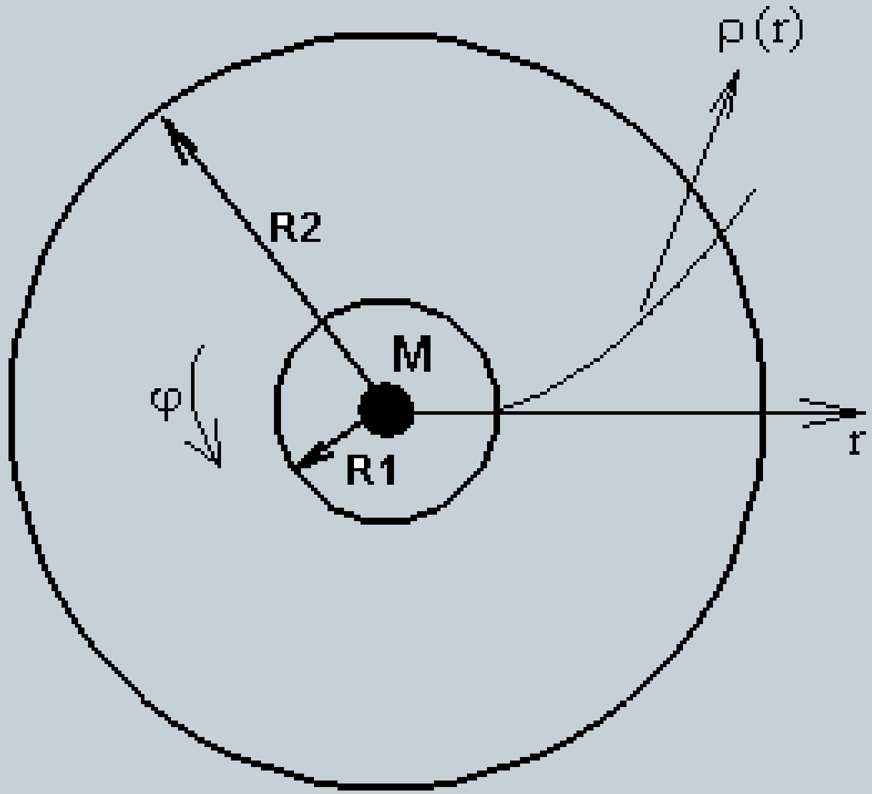
LOCAL DISTURBANCE.
ANGULAR MOMENTUM REDISTRIBUTION



Angular momentum redistribution along the radius at time moments

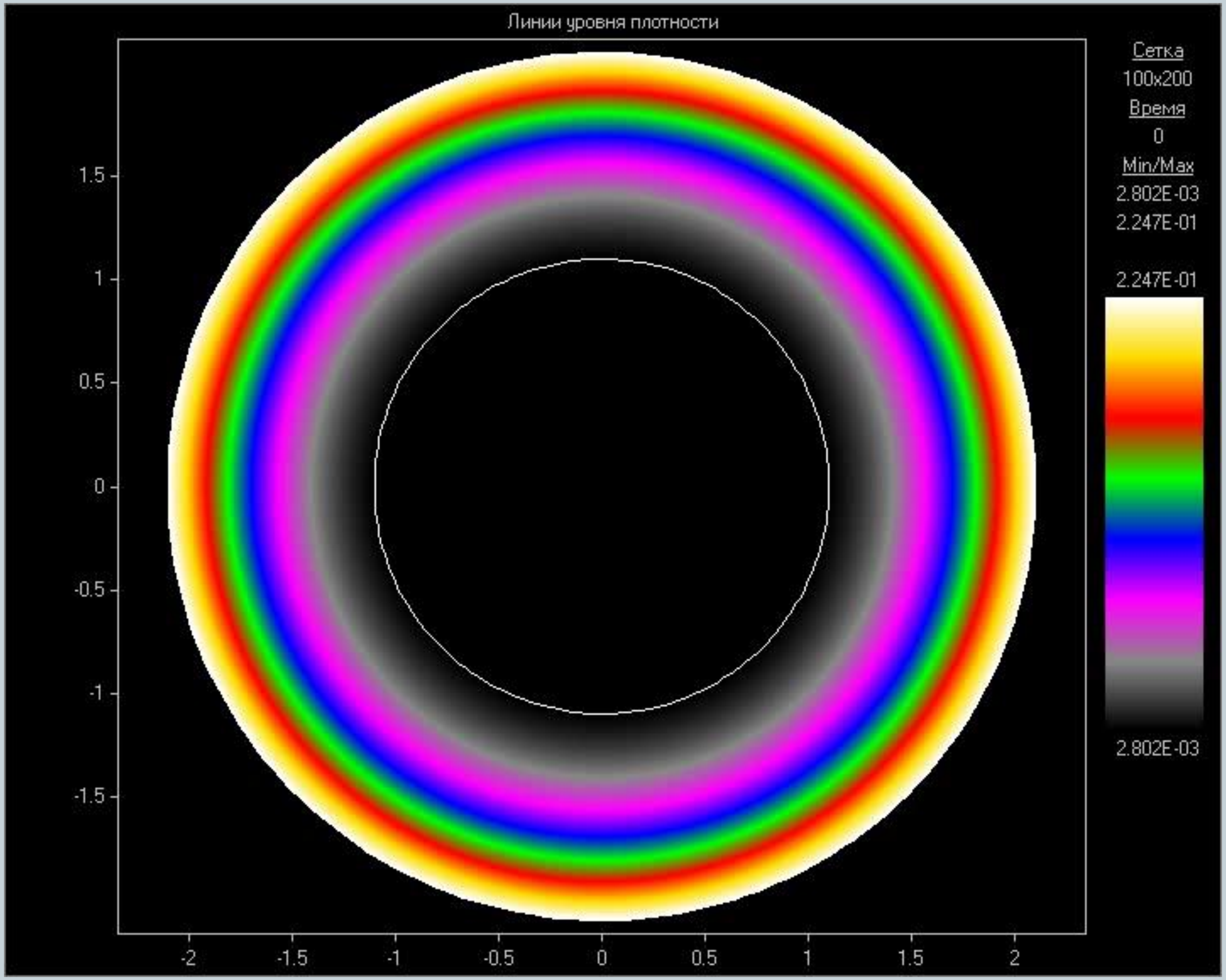
$t_1=0$ (——) и $t_2 = 5$ rev (—— - $\varphi=0°$, ●●● - $\varphi=180°$)

KEPLERIAN ACCRETION DISK

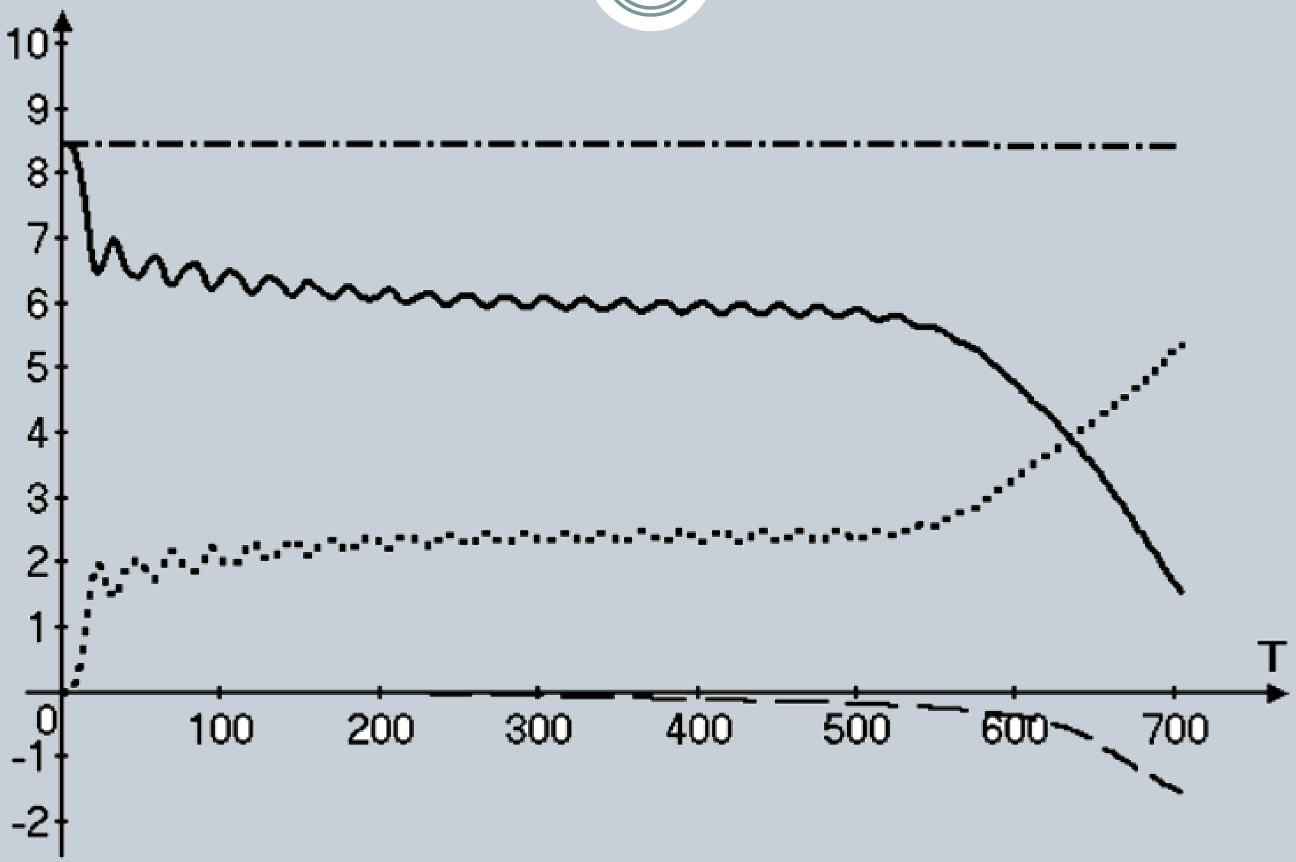


$$v = v_0 \sqrt{1/r}$$

PERTURBATIONS OF THE AZIMUTHAL VELOCITY IN THE ZONE OF MAXIMUM DENSITY

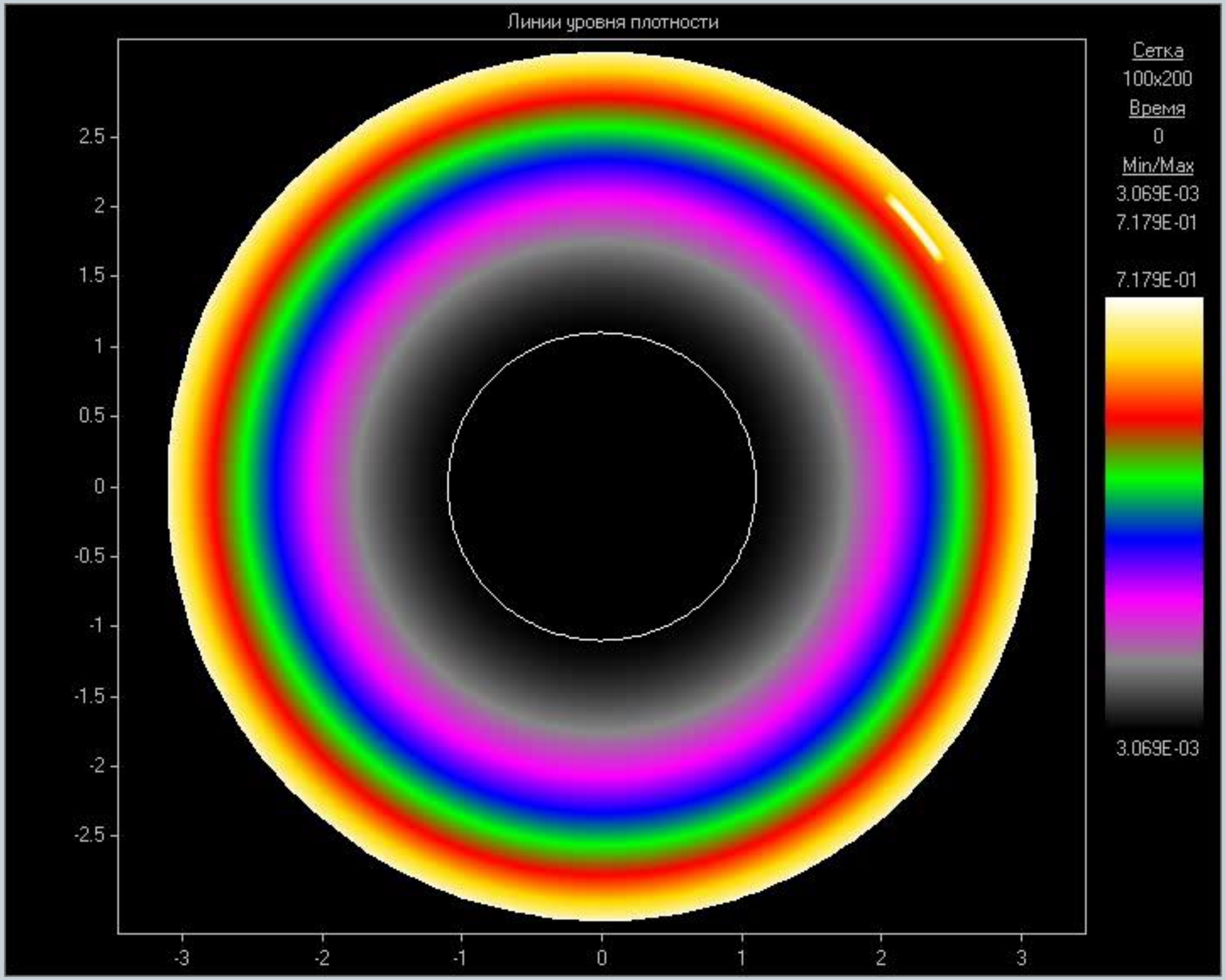


BEHAVIOR OF THE MATTER MASS IN KEPLERIAN ACCRETION DISKS

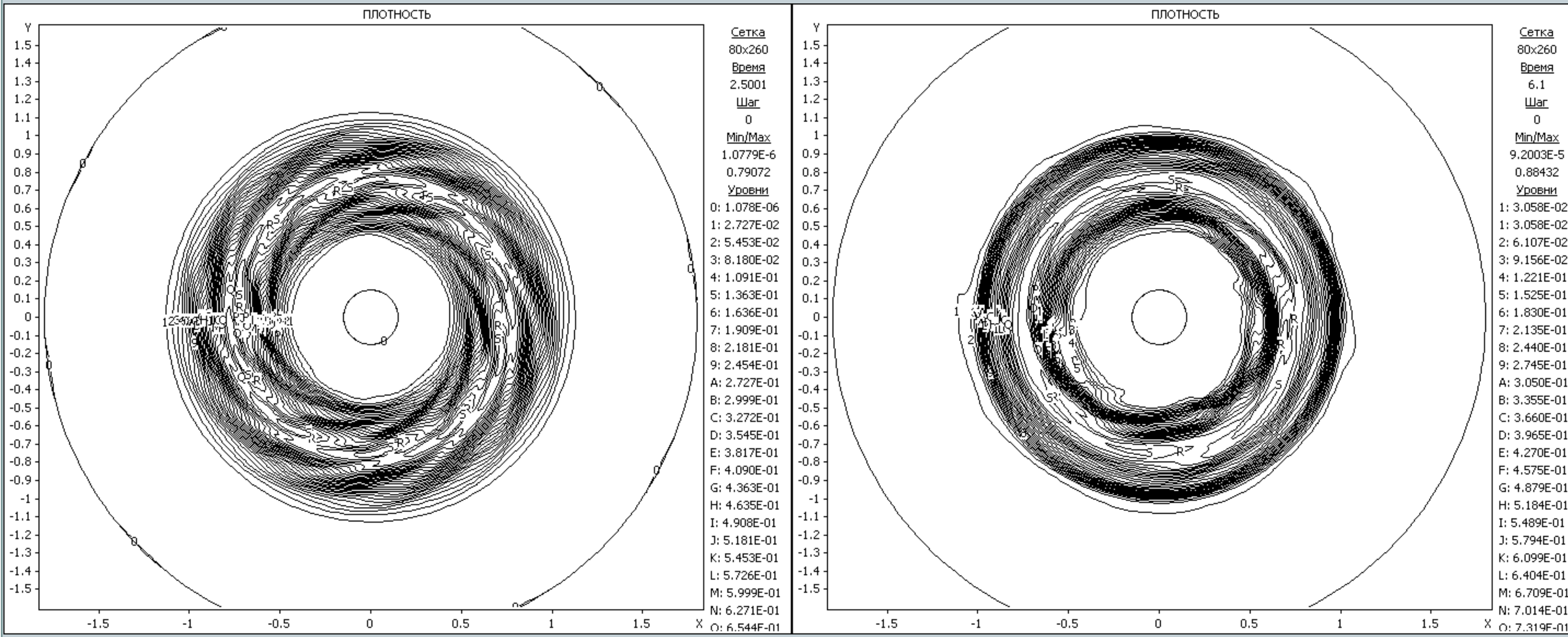


Mass of the matter in computational domain (——), the mass flow through the outer boundary (●●●), the mass flow through the inner boundary (— —) and the sum of the lost mass and mass in the computational domain (— • —).

DISTURBANCE IN THE OUTER PART OF THE KEPLERIAN ACCRETION DISK



3D MODELING



ПЛОТНОСТЬ
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Шаг
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Min/Max
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0.79072
Уровни
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2: 5.453E-02
3: 8.180E-02
4: 1.091E-01
5: 1.363E-01
6: 1.636E-01
7: 1.909E-01
8: 2.181E-01
9: 2.454E-01
A: 2.727E-01
B: 2.999E-01
C: 3.272E-01
D: 3.545E-01
E: 3.817E-01
F: 4.090E-01
G: 4.363E-01
H: 4.635E-01
I: 4.908E-01
J: 5.181E-01
K: 5.453E-01
L: 5.726E-01
M: 5.999E-01
N: 6.271E-01
O: 6.544E-01
ПЛОТНОСТЬ
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Время
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Шаг
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Min/Max
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0.88432
Уровни
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1: 3.058E-02
2: 6.107E-02
3: 9.156E-02
4: 1.221E-01
5: 1.525E-01
6: 1.830E-01
7: 2.135E-01
8: 2.440E-01
9: 2.745E-01
A: 3.050E-01
B: 3.355E-01
C: 3.660E-01
D: 3.965E-01
E: 4.270E-01
F: 4.575E-01
G: 4.879E-01
H: 5.184E-01
I: 5.489E-01
J: 5.794E-01
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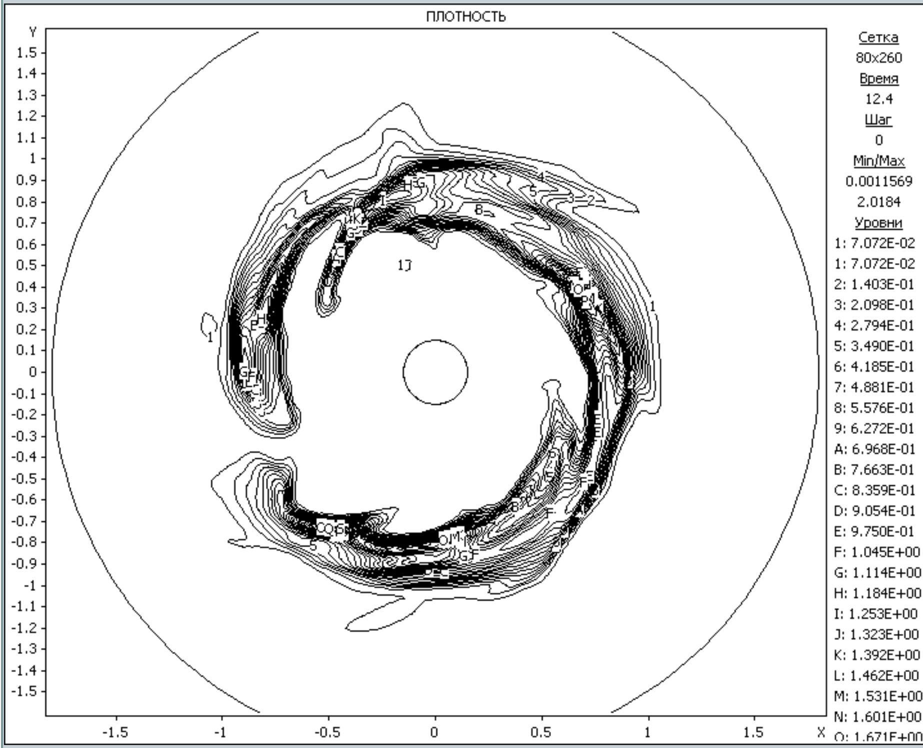
M: 6.709E-01
N: 7.014E-01
O: 7.319E-01

3D MODELING



ПЛОТНОСТЬ
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Шаг
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Min/Max
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Уровни
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4: 2.794E-01
5: 3.490E-01
6: 4.185E-01
7: 4.881E-01
8: 5.576E-01
9: 6.272E-01
A: 6.968E-01
B: 7.663E-01
C: 8.359E-01
D: 9.054E-01
E: 9.750E-01
F: 1.045E+00
G: 1.114E+00
H: 1.184E+00
I: 1.253E+00
J: 1.323E+00
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L: 1.462E+00
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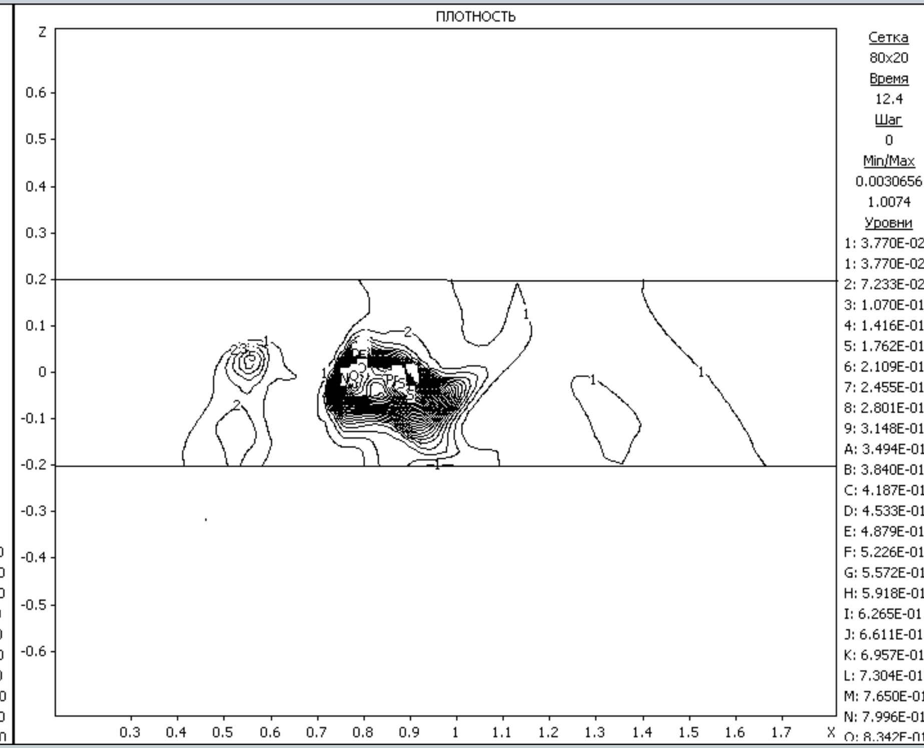
N: 1.601E+00
O: 1.671E+00
Y
X



ПЛОТНОСТЬ
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Время
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Шаг
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Min/Max
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1.0074
Уровни
1: 3.770E-02
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2: 7.233E-02
3: 1.070E-01
4: 1.416E-01
5: 1.762E-01
6: 2.109E-01
7: 2.455E-01
8: 2.801E-01
9: 3.148E-01
A: 3.494E-01
B: 3.840E-01
C: 4.187E-01
D: 4.533E-01
E: 4.879E-01
F: 5.226E-01
G: 5.572E-01
H: 5.918E-01
I: 6.265E-01
J: 6.611E-01
K: 6.957E-01
L: 7.304E-01
M: 7.650E-01
N: 7.996E-01
O: 8.342E-01
Z
X

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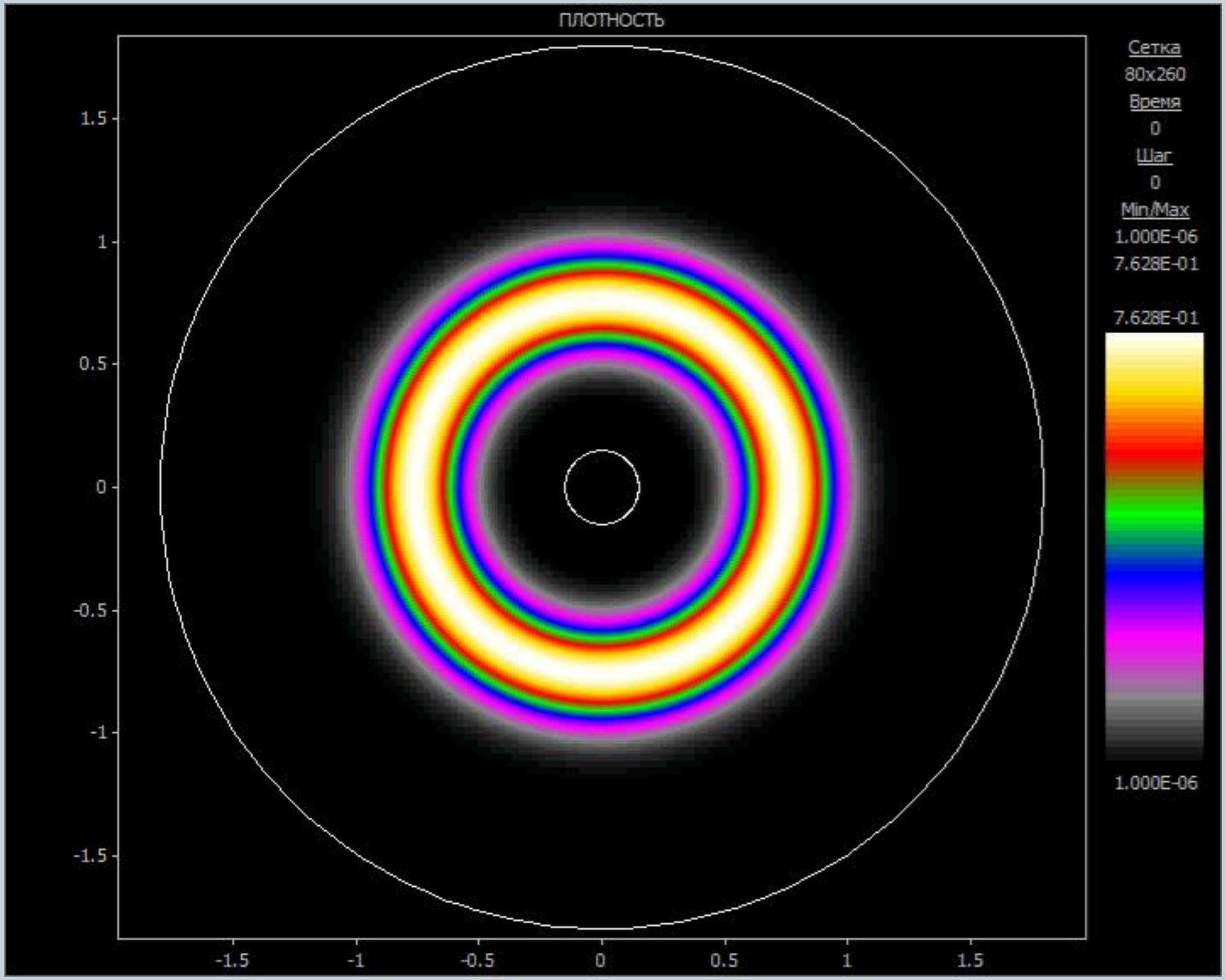
3D MODELING



ПЛОТНОСТЬ
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Шаг
0
Min/Max
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7.628E-01
7.628E-01
1.000E-06
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5

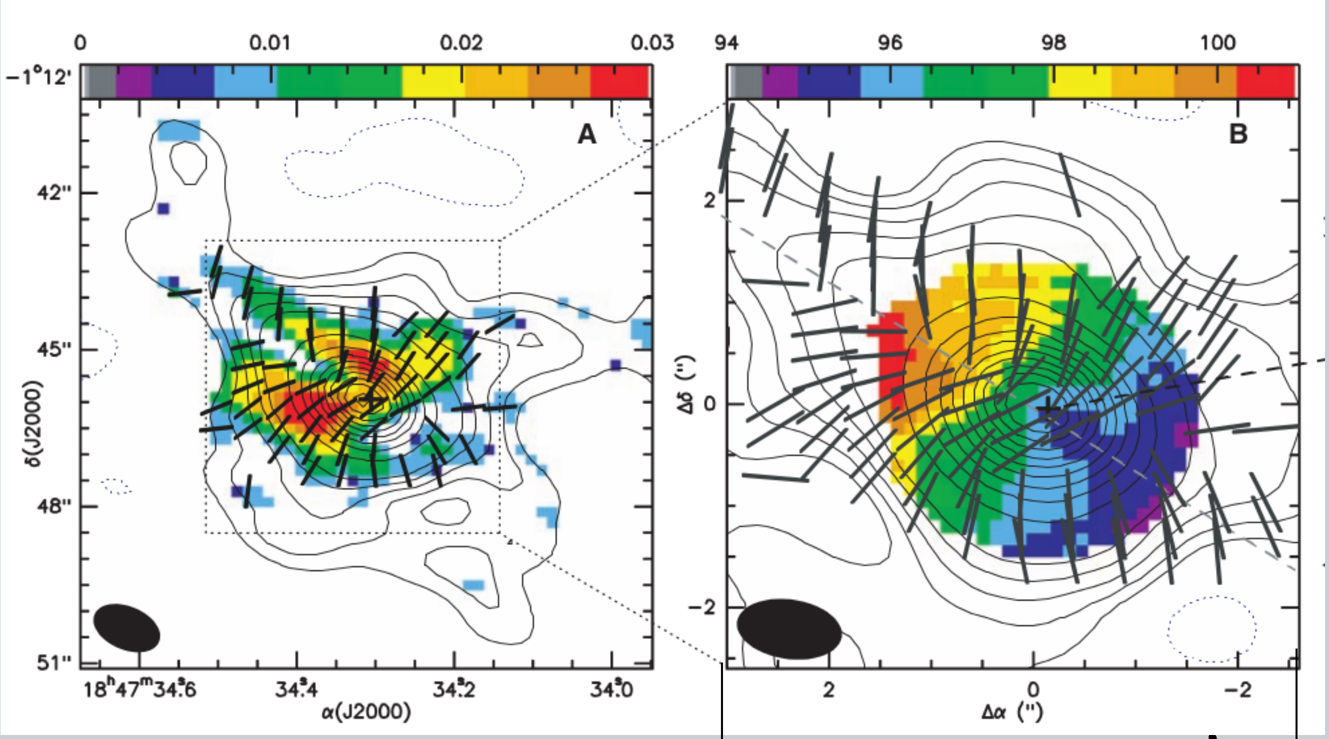
CONCLUSIONS

1. Large-scale vortex flows can arise in stellar accretion disks.
2. Large-scale vortexes could transfer the angular momentum to the outer parts of the accretion stellar disk without heating of the matter.
3. 3D approach give the similar to 2D approach results at initial stage of the evolution.

MHD APPROACH

Massive star-forming region G31.41

[Science **324**, 1408 (2009)]



Distance 7900 pc

~ 0.2 pc ~ 40 000 au

IDEAL MHD EQUATIONS

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_z}{\partial z} = 0$$ - mass conservation law

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_j}\left(v_i v_j - B_i B_j + \delta_{ij} P\right) = 0$$ - momentum conservation law

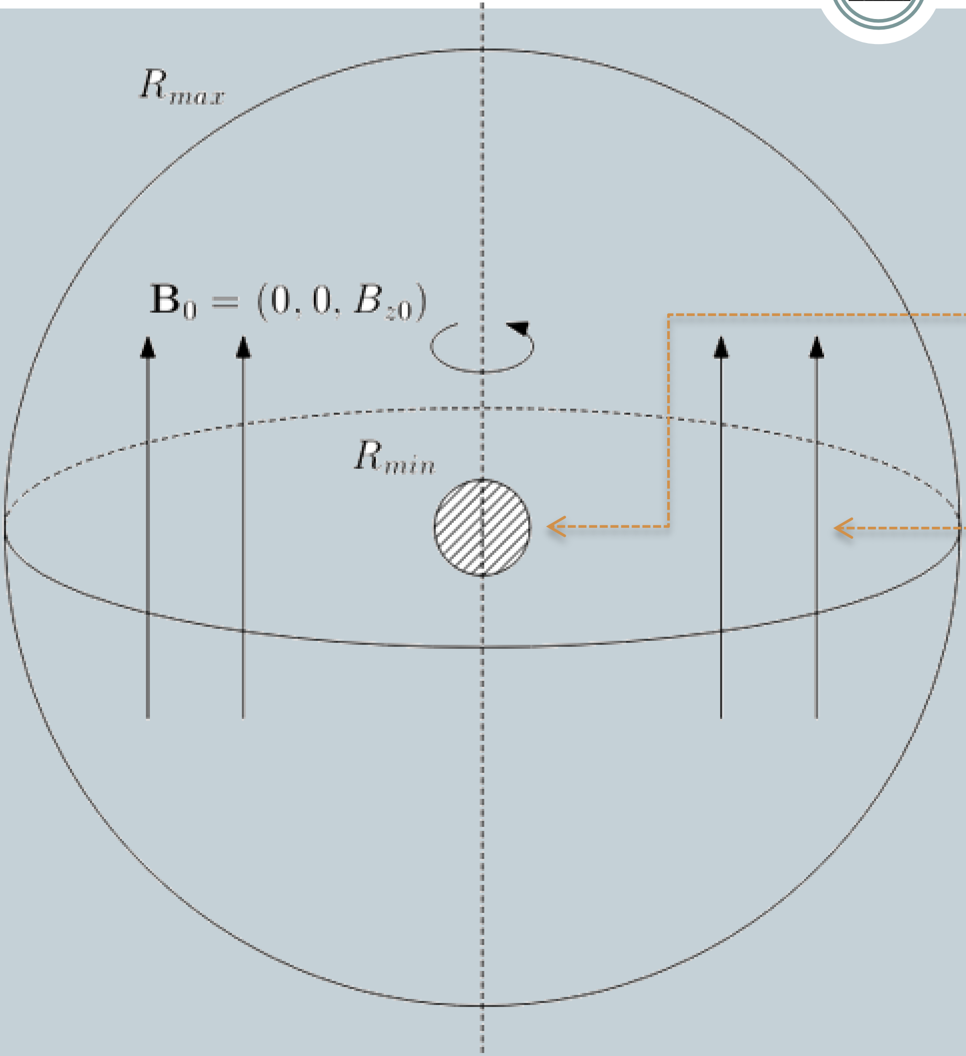
$$\frac{\partial B_i}{\partial t} + \frac{\partial}{\partial x_j}\left(B_i v_j - B_j v_i\right) = 0$$ - magnetic field induction

$$P = c_T^2 \rho + \frac{\mathbf{B}^2}{2}$$ - pressure equation in the isothermal approximation

Notations:

- ρ – density;
- v_x, v_y, v_z – velocity vector components;
- $\mathbf{B} = (B_x, B_y, B_z)$ – magnetic induction vector and its components
- c_T - isothermal speed of sound
- $i, j = \{x, y, z\}$ – axis subscripts

NUMERICAL MODEL



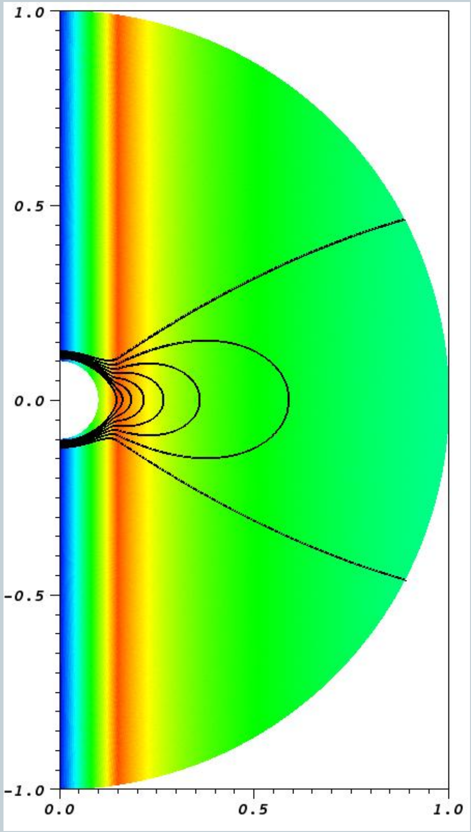
Computational area located between two spheres

Gravitating object located in the center

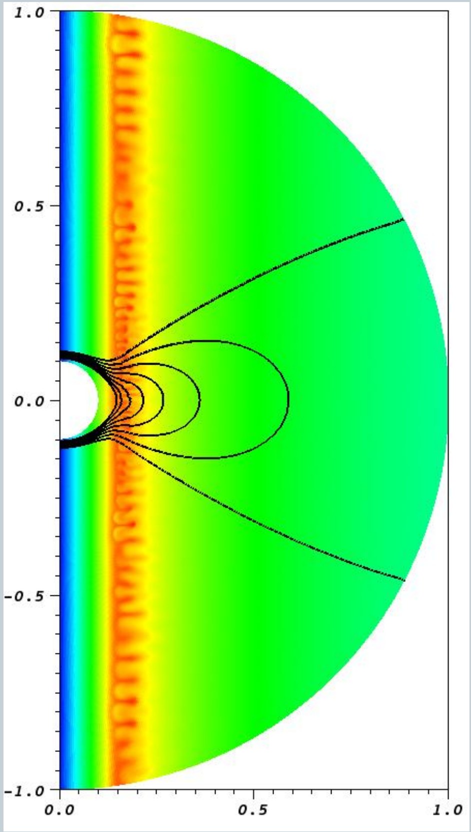
Initial uniform magnetic field

INITIAL STAGE OF INSTABILITY DEVELOPMENT. 2D MODEL

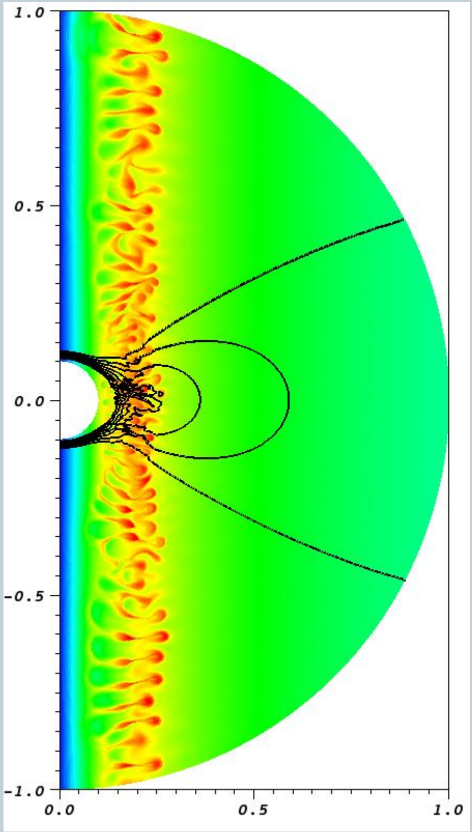
Color plot denotes azimuthal speed v_φ, density ρ is plotted with lines.



$t = 0$



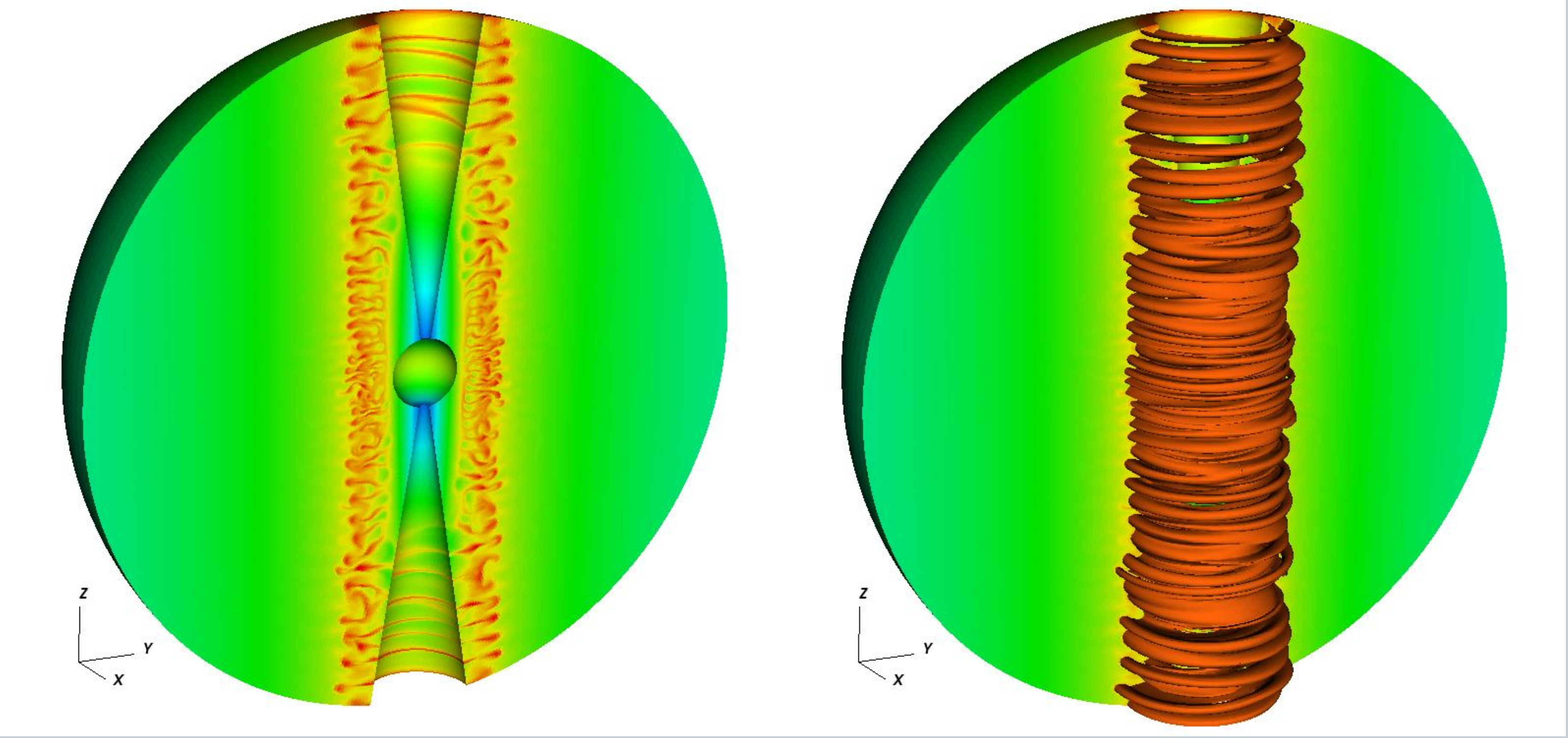
$t = 2$ revolutions



$t = 3$ revolutions

INITIAL STAGE OF INSTABILITY DEVELOPMENT. 3D MODEL

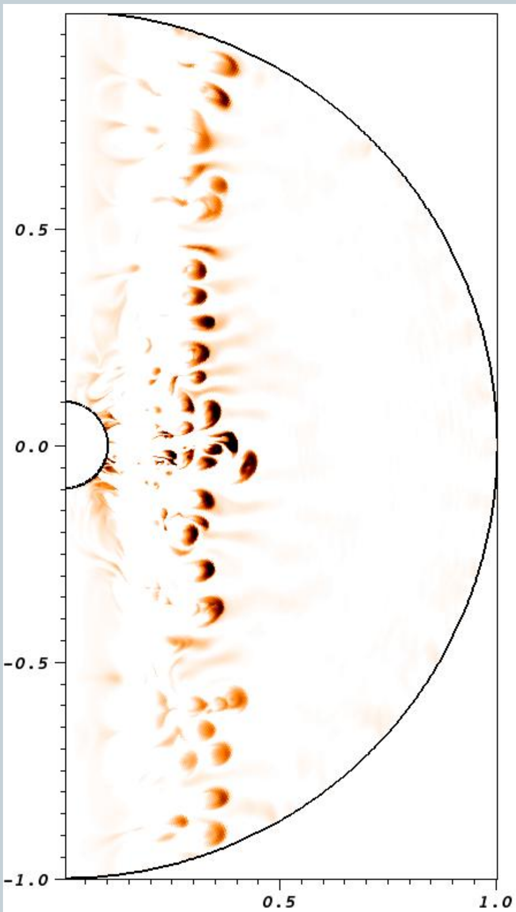
Azimuthal speed (color plot and isosurfaces)



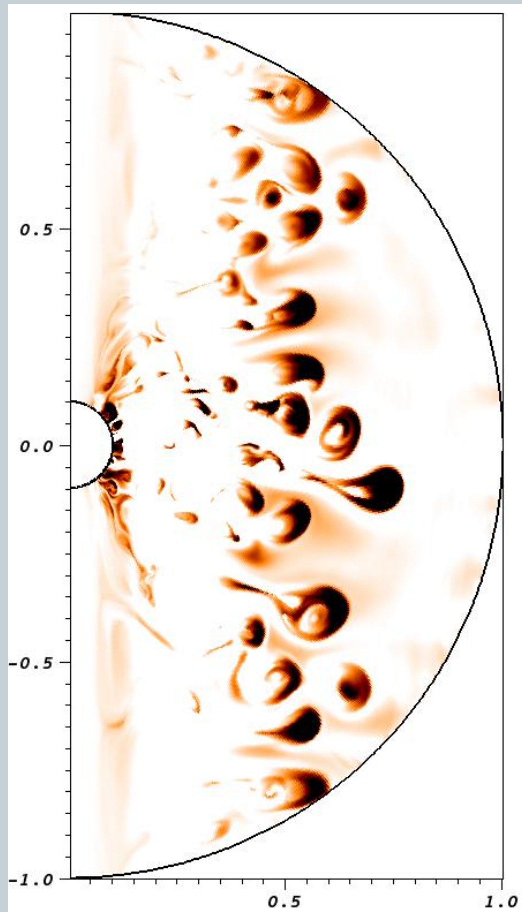
$t = 3$ revolutions

ANGULAR MOMENTUM TRANSFER

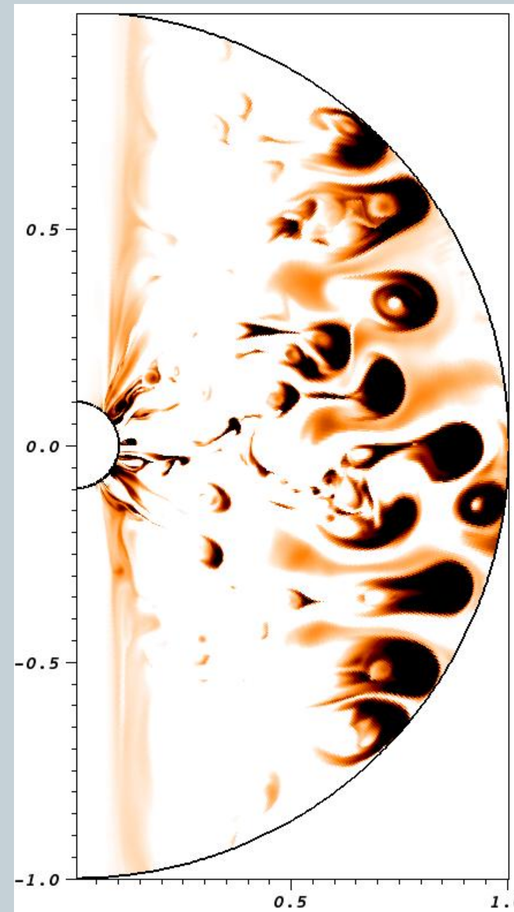
Dark color denotes positive angular momentum fluctuations



5 revolutions



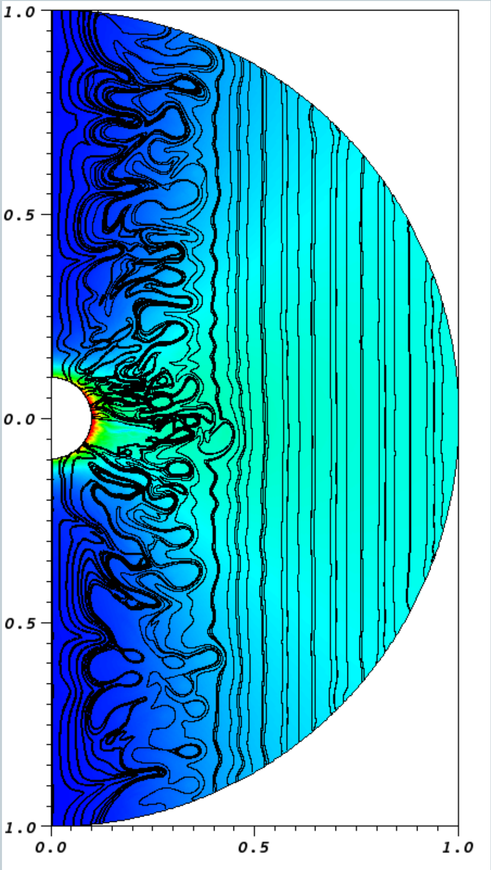
10 revolutions



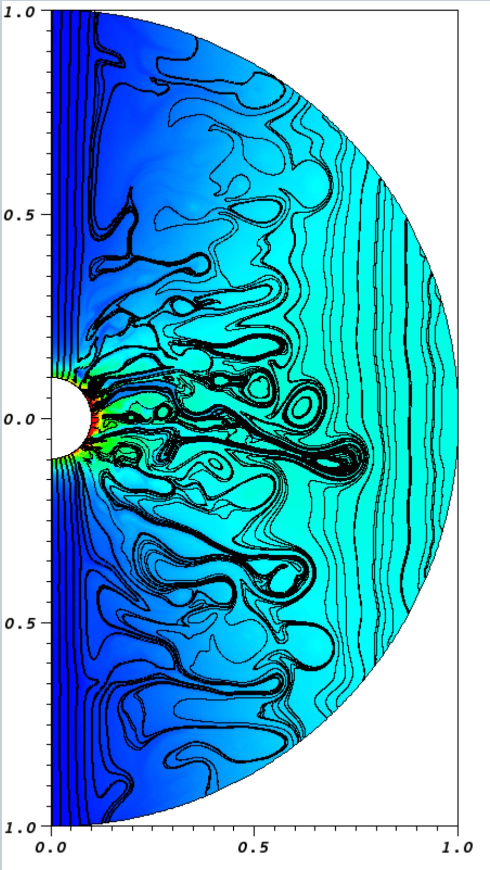
15 revolutions

MAGNETIC FIELD

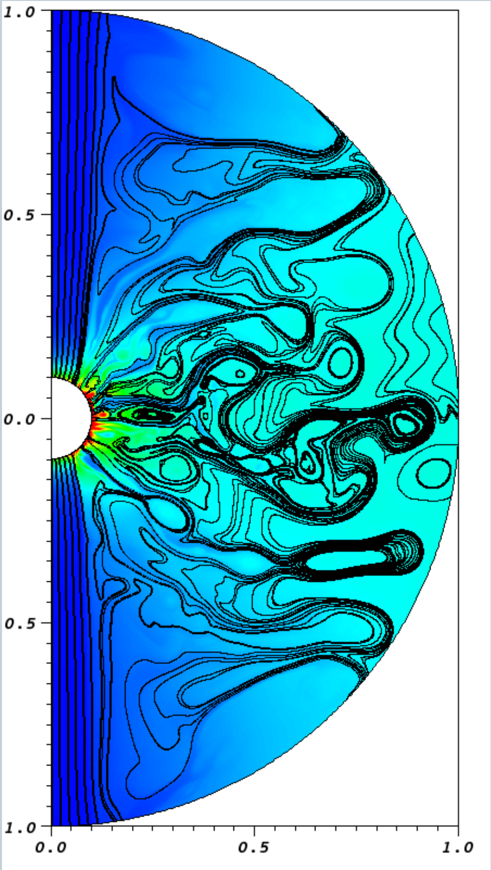
Magnetic field lines is plotted at the top of the color plot of the density



5 revolutions



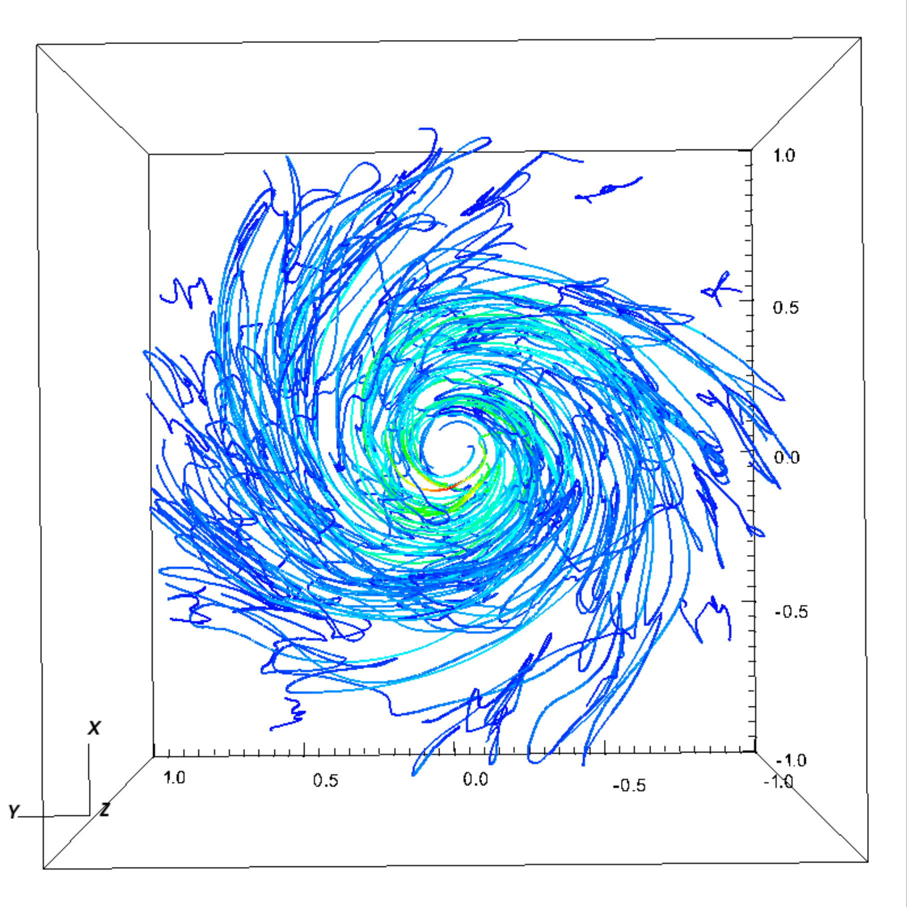
10 revolutions



15 revolutions

3D PLOT OF MAGNETIC FIELD LINES

rotation axis is z

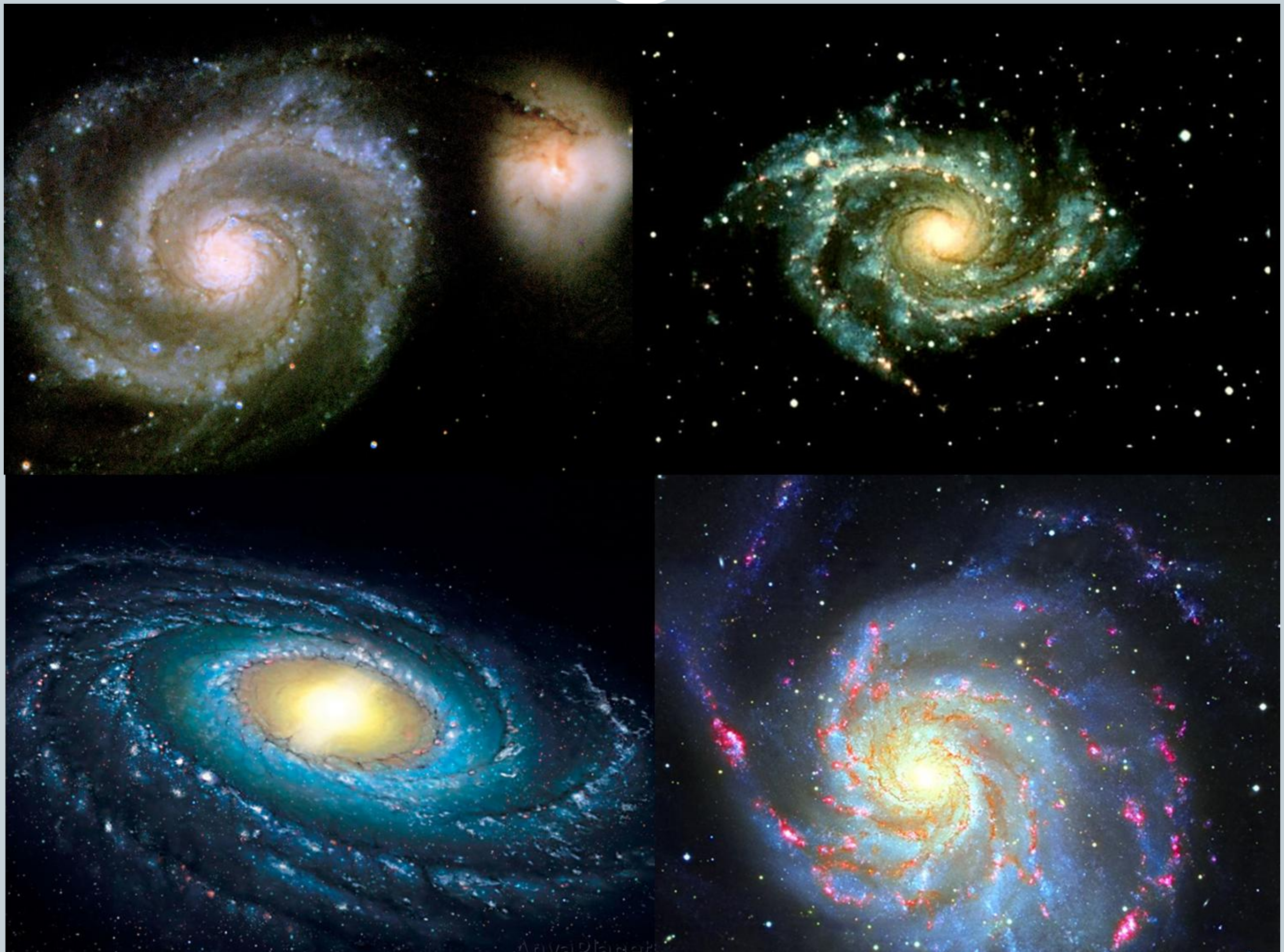


$t = 15$ revolutions

CONCLUSIONS

1. The possibility of the instability development in the presence of a weak magnetic field is shown.
2. It is shown that the development of instability leads to a transport of angular momentum to the disk periphery by large-scale vortex structures together with the accretion of matter onto the gravitating object.
3. The accretion rate and magnetic-field lines near the equator are in agreement with observations.

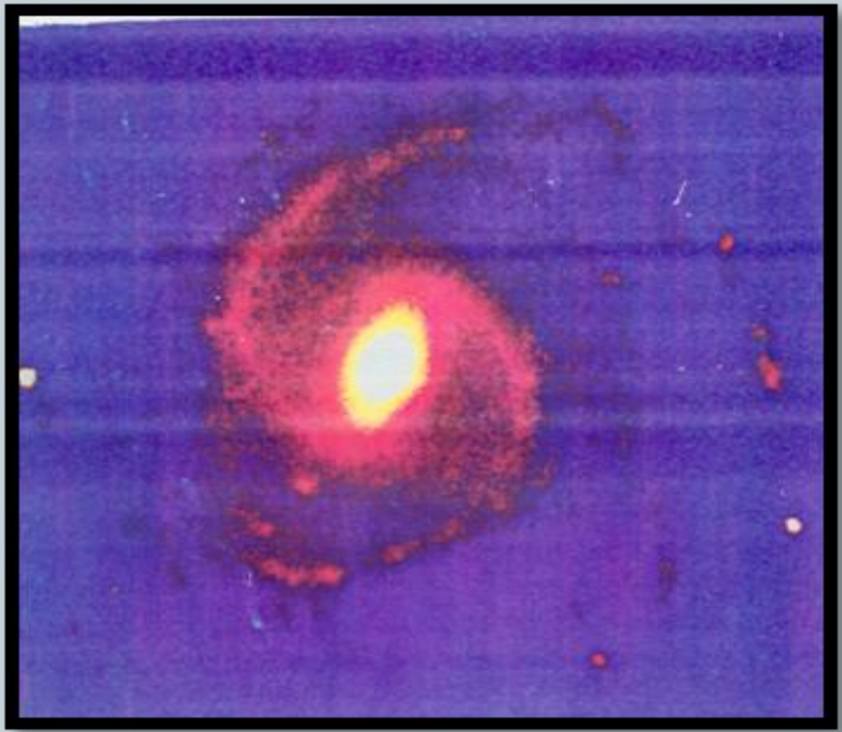
GALAXY DISK MODELING



MAIN IDEA

The origin and formation in a two-armed global morphology of spiral galaxies the number of gas-dynamic elements similar to each other in form and independent in brightness.

MULTI-ARMED STRUCTURES: OBSERVATIONS



Galaxy NGC 309

"… it is at first difficult to believe that one is not looking at the same galaxy."

D. L. Block and R. J. Wainscoat, Nature **353**, 48 (1991).

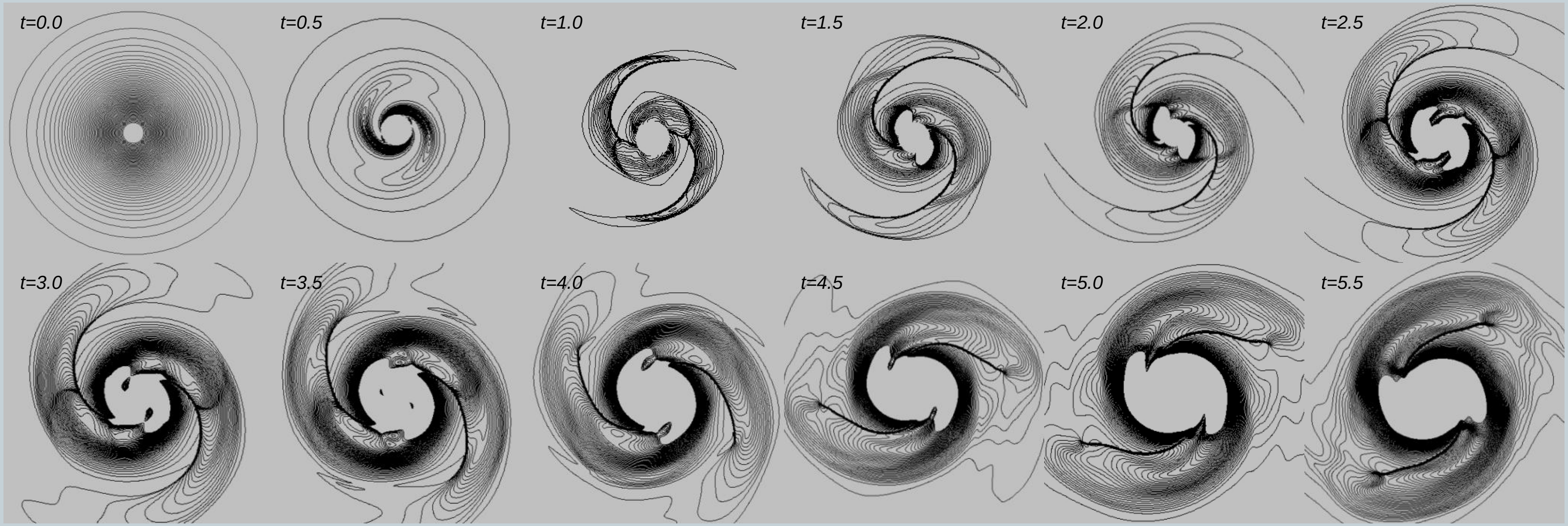
GRAVITY

$$\varphi = \varphi_H + \varphi_s + \tilde{\varphi},$$

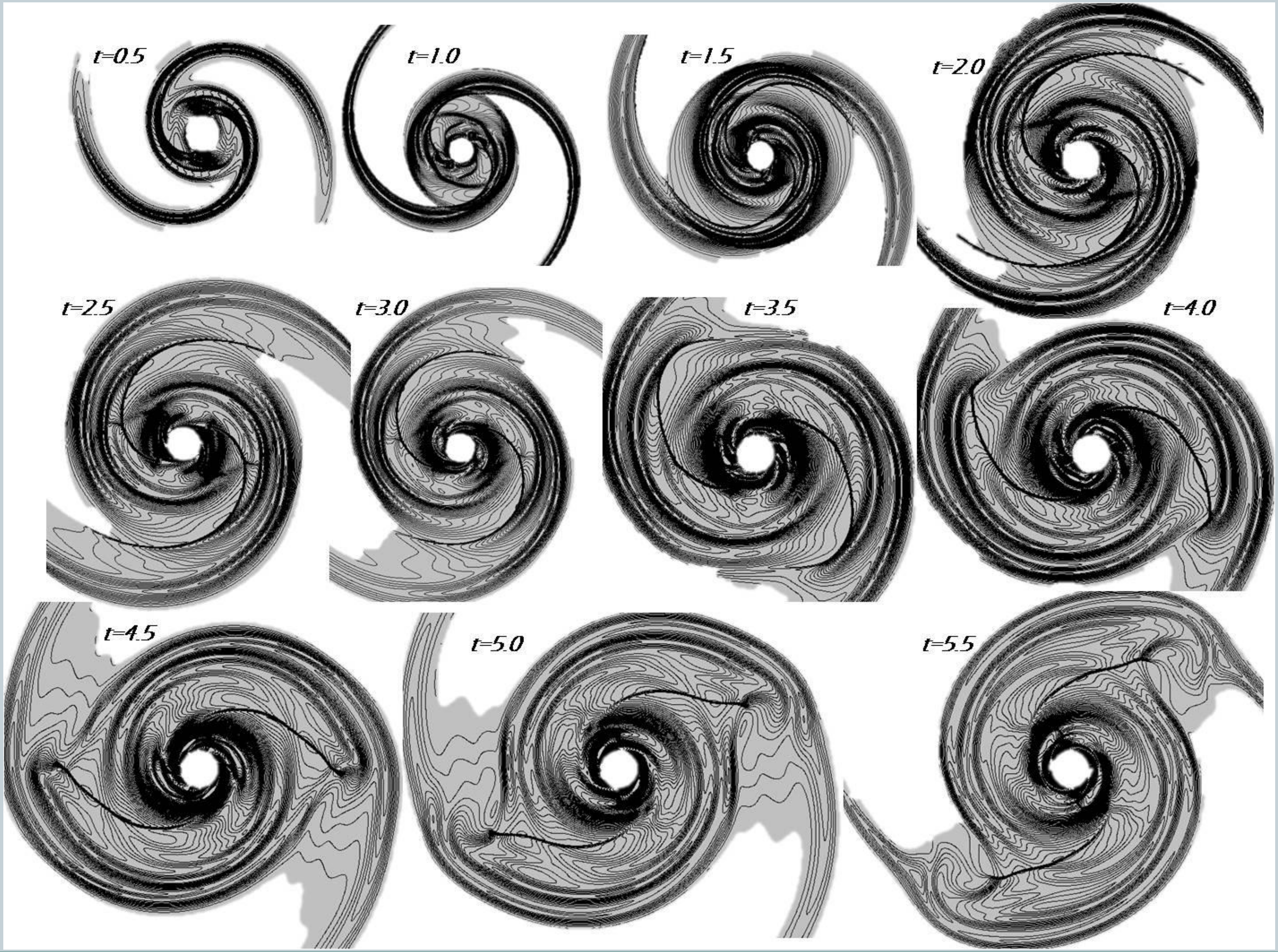
$\varphi_H + \varphi_s$ - axisymmetric part (halo, bulge, stellar disk)

$\tilde{\varphi}$ - non-axisymmetric part (spiral star arms)

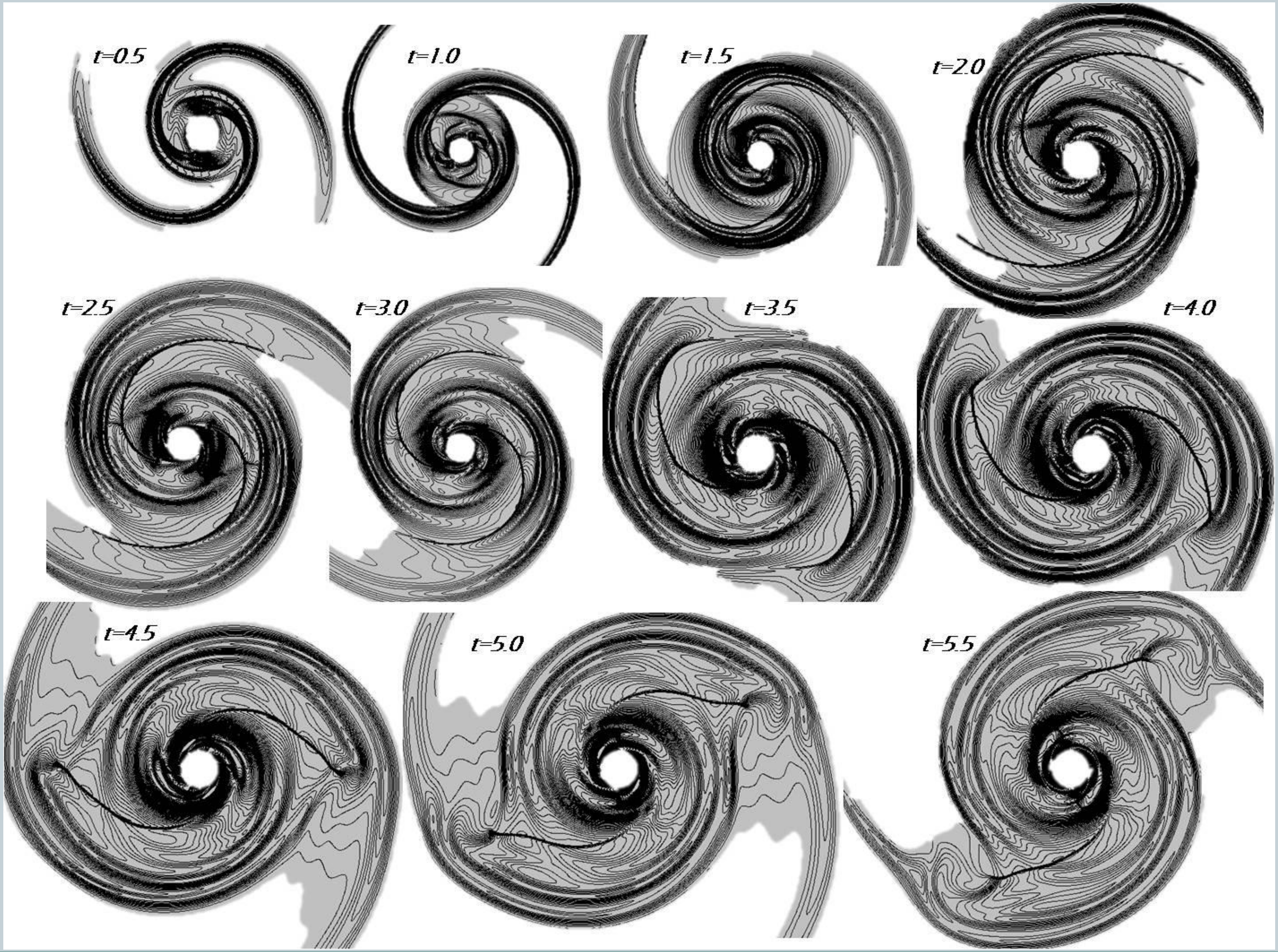
STAGES OF SPIRAL PATTERN FORMATION. ISOLINES OF THE GAS PRESSURE IN TIME



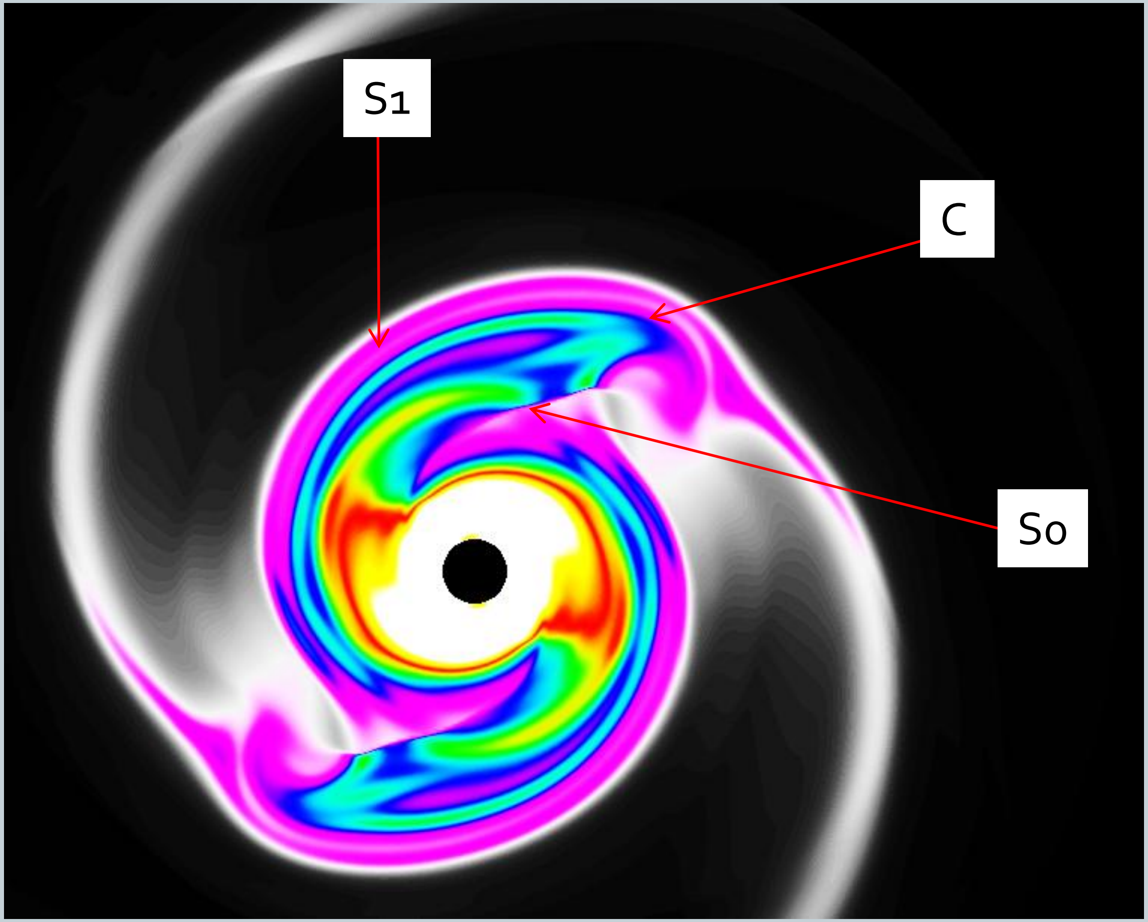
STAGES OF SPIRAL PATTERN FORMATION. ISOLINES OF THE GAS DENSITY IN TIME



STAGES OF SPIRAL PATTERN FORMATION. ISOLINES OF THE GAS DENSITY IN TIME

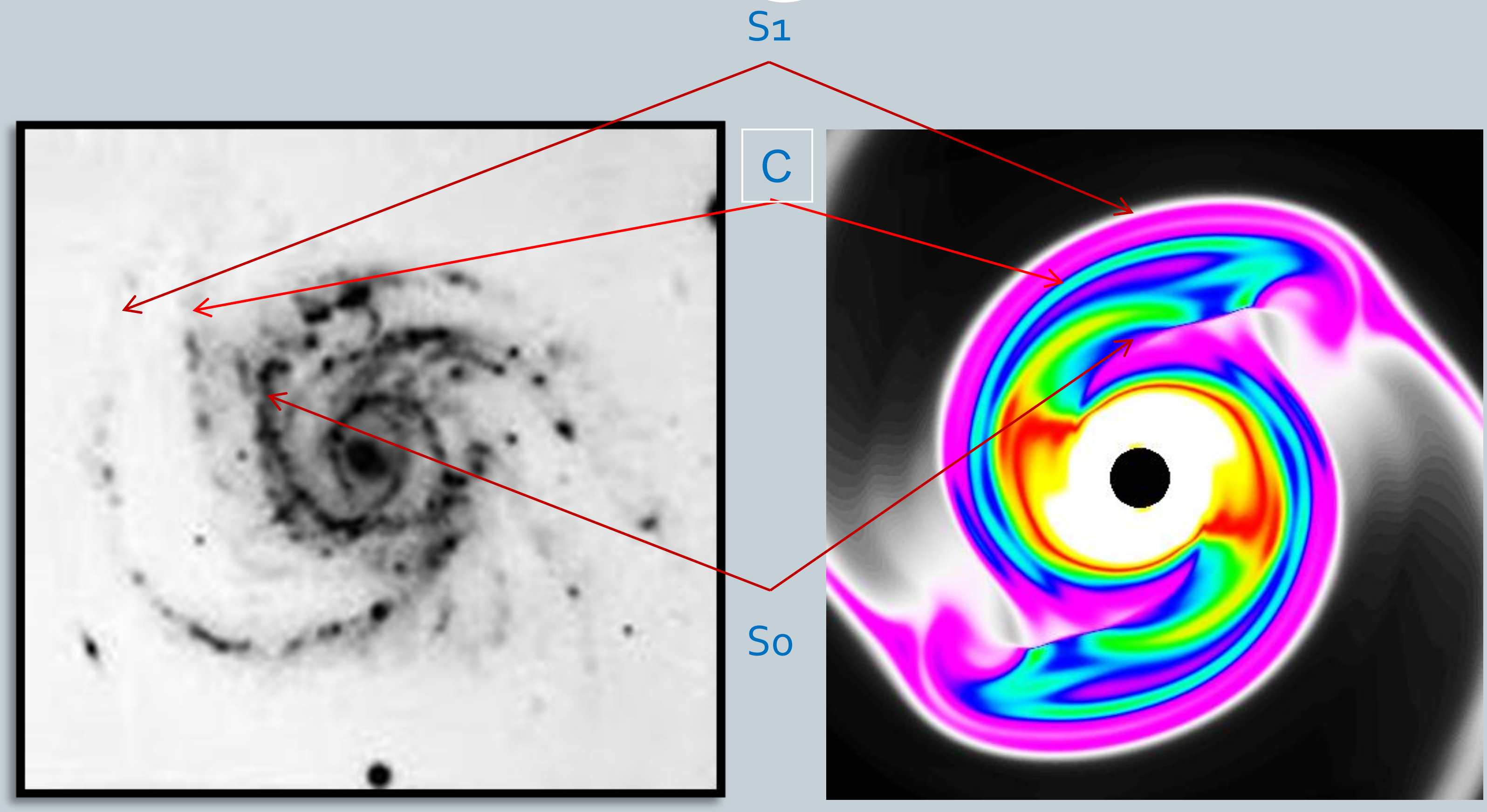


MULTI-ARMED GAS CONFIGURATION IN A TWO-ARMED SPIRAL GRAVITATIONAL FIELD



S0, S1 – shock waves, C – contact discontinuity

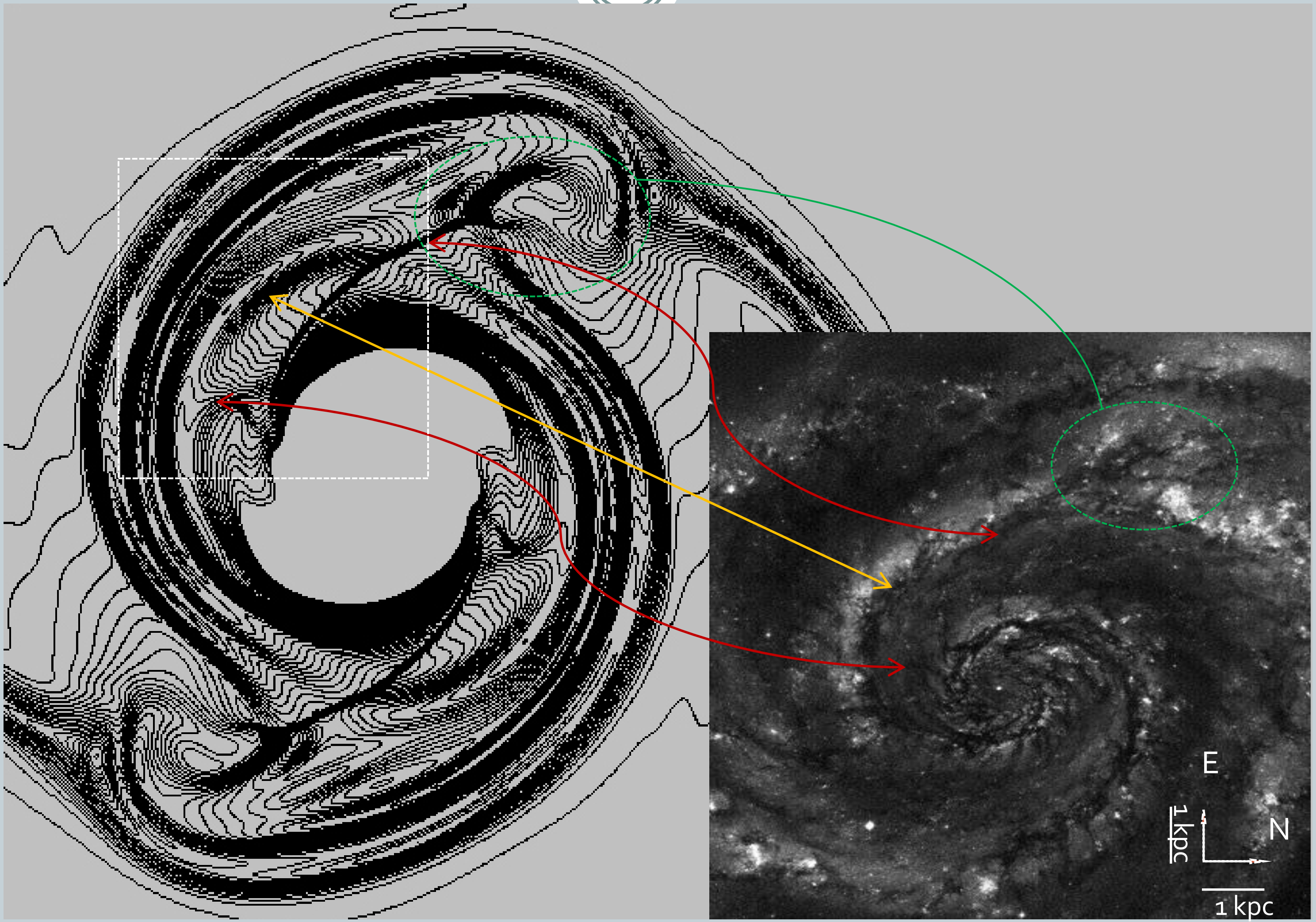
MULTI-ARMED STRUCTURES: COMPARISONS



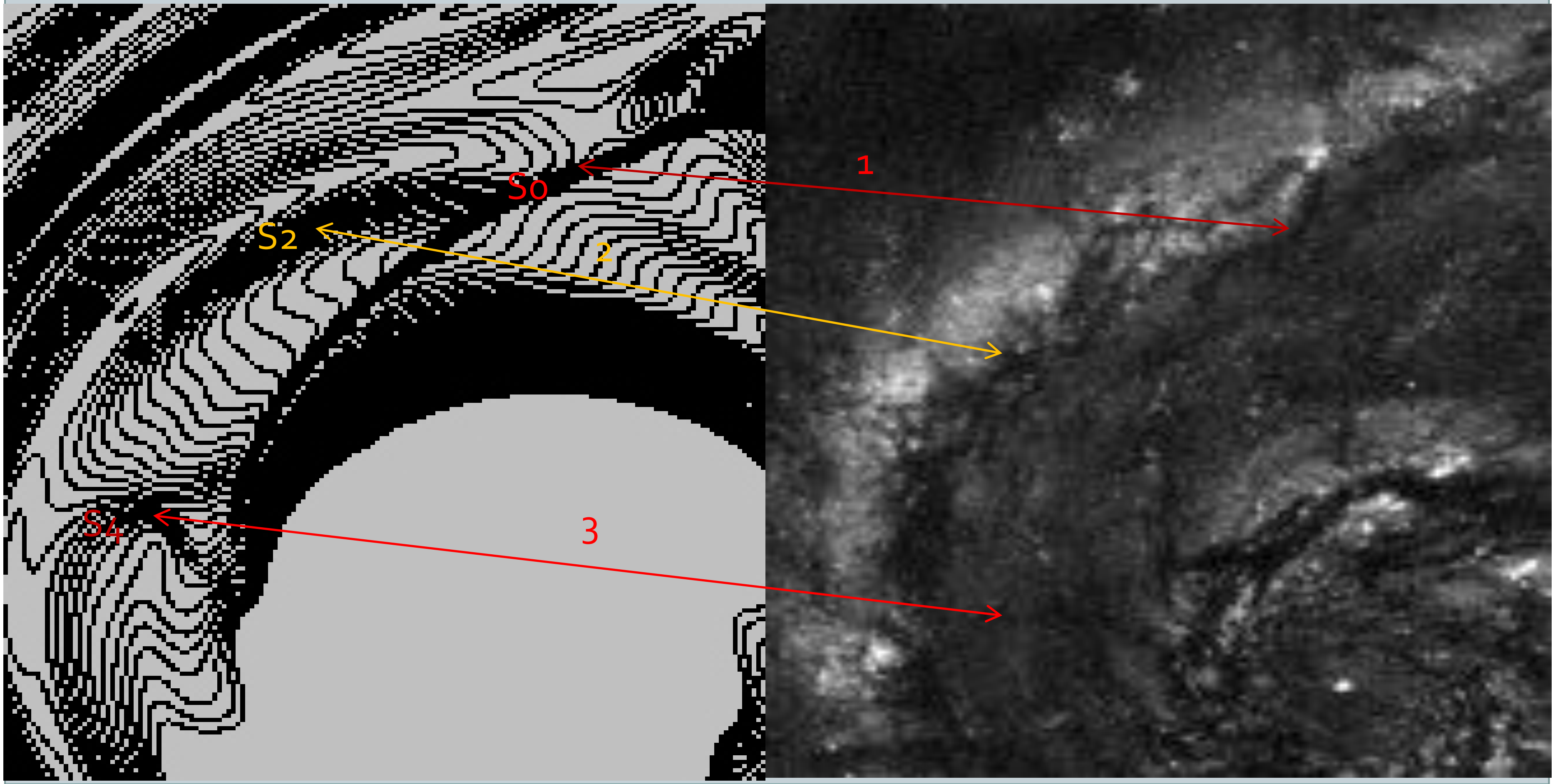
NGC 309

COMPARISON WITH OBSERVATIONS

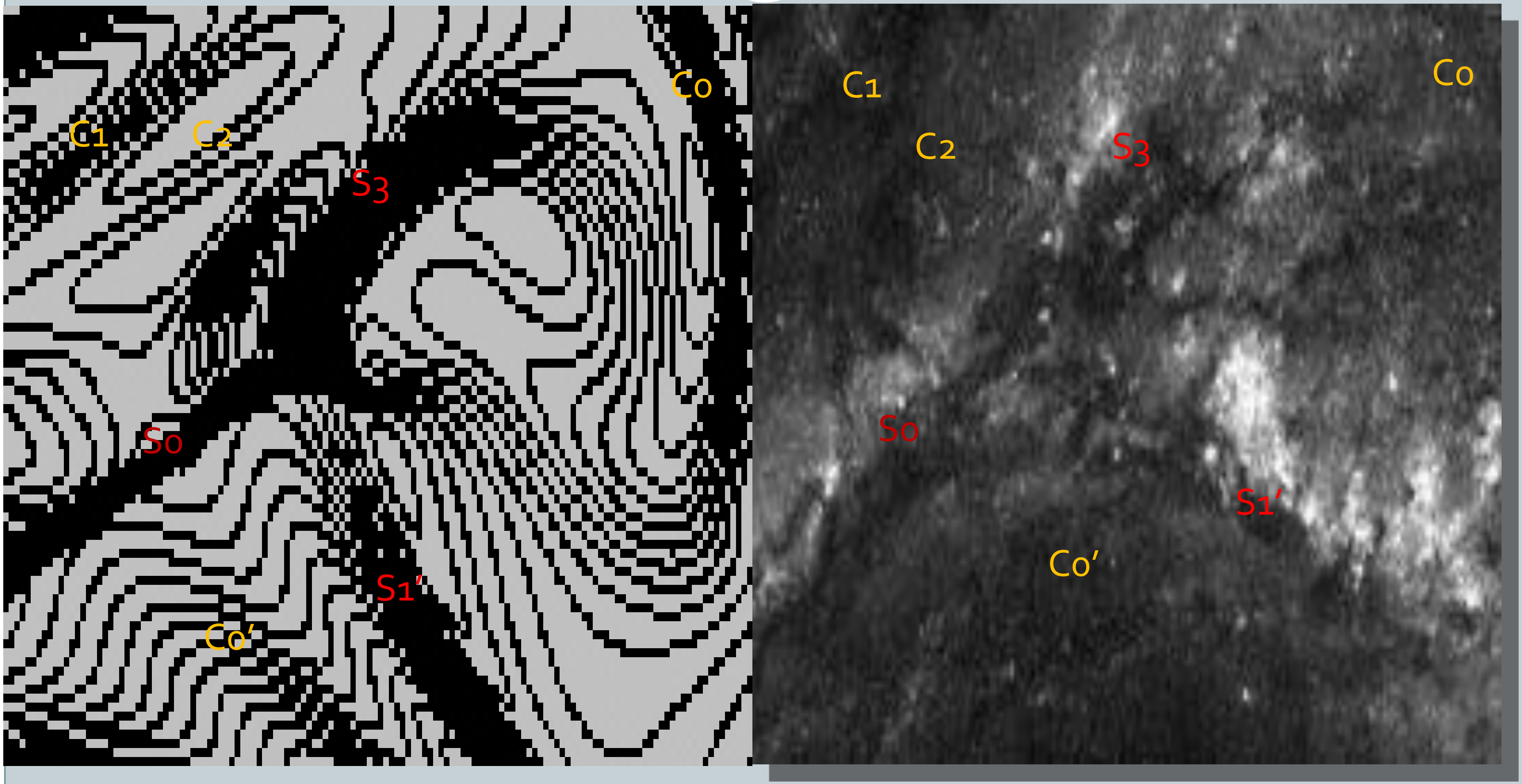
t=5.7



COMPARISON WITH OBSERVATIONS



COMPARISON WITH OBSERVATIONS



CONCLUSIONS

1. Gasdynamical discontinuities arised in the flow of matter in the two-armed spiral galaxy could change the morphology to multi-armed.
2. The results are in good agreement with observations.

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SUPERCOMPUTER MATHEMATICAL MODELLING OF MATTER FLOWS IN ACCRETION STELLAR DISKS

Thanks for Listening

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